The Science of Complex Networks and the Internet:

Lies, Damned Lies, and Statistics

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Today's Agenda

- The case of high-quality Internet data (traffic)
- The case of low-quality Internet data (connectivity)
  - More "normal" than Normal: De-mystifying power-laws
  - How to make everything look like a power-law
- Internet modeling
  - Beyond traditional approaches
  - From data-fitting to reverse-engineering
- An engineering perspective to modeling highly engineered systems
  - Recognize the limitations of the available data
  - Recognize the power of domain knowledge
  - Model validation and reverse-engineering

Analysis of Internet Data: Know your Statistics!

Analyzing High-Quality Internet Data

February 23, 2010

Internet Traffic Measurements: Fact 1

- Early example of measurement-driven Internet research
  - What does real Internet traffic look like?
  - Answer: Go and measure the traffic!
- Traffic data collection
  - Need special-purpose hardware
  - Enormous efforts to check quality of measurements
  - “Measuring the measurer”
- Implications
  - Can get as much high-quality data as one wants
  - Limited by data storage, processing, and analysis capabilities

Internet Traffic Measurements: Fact 2

- High-quality data sets
  - The collection hardware/software has been extensively tested
- High-volume data sets
  - Individual data sets are huge
  - Huge number of different data sets
  - Even more and different data in the future
- Rich semantic content
  - Each measurement contains lots of information
  - IP packet: more than just arrival time and size
  - IP flow: rich source of information
  - User session: more than arrival time and duration

Internet Traffic Measurements: Fact 3

- High variability everywhere you look
  - Link bandwidth: Kbps – Gbps
  - File sizes: a few bytes – Mega/Gigabytes
  - Flows: a few packets – 100,000+ packets
  - Delay: Milliseconds – seconds and beyond
  - etc.
- Statistical dilemma
  - High variability: Large, but finite variance?
  - High variability: Infinite variance as mathematical abstraction

On Traditional Analysis of Internet Traffic Data

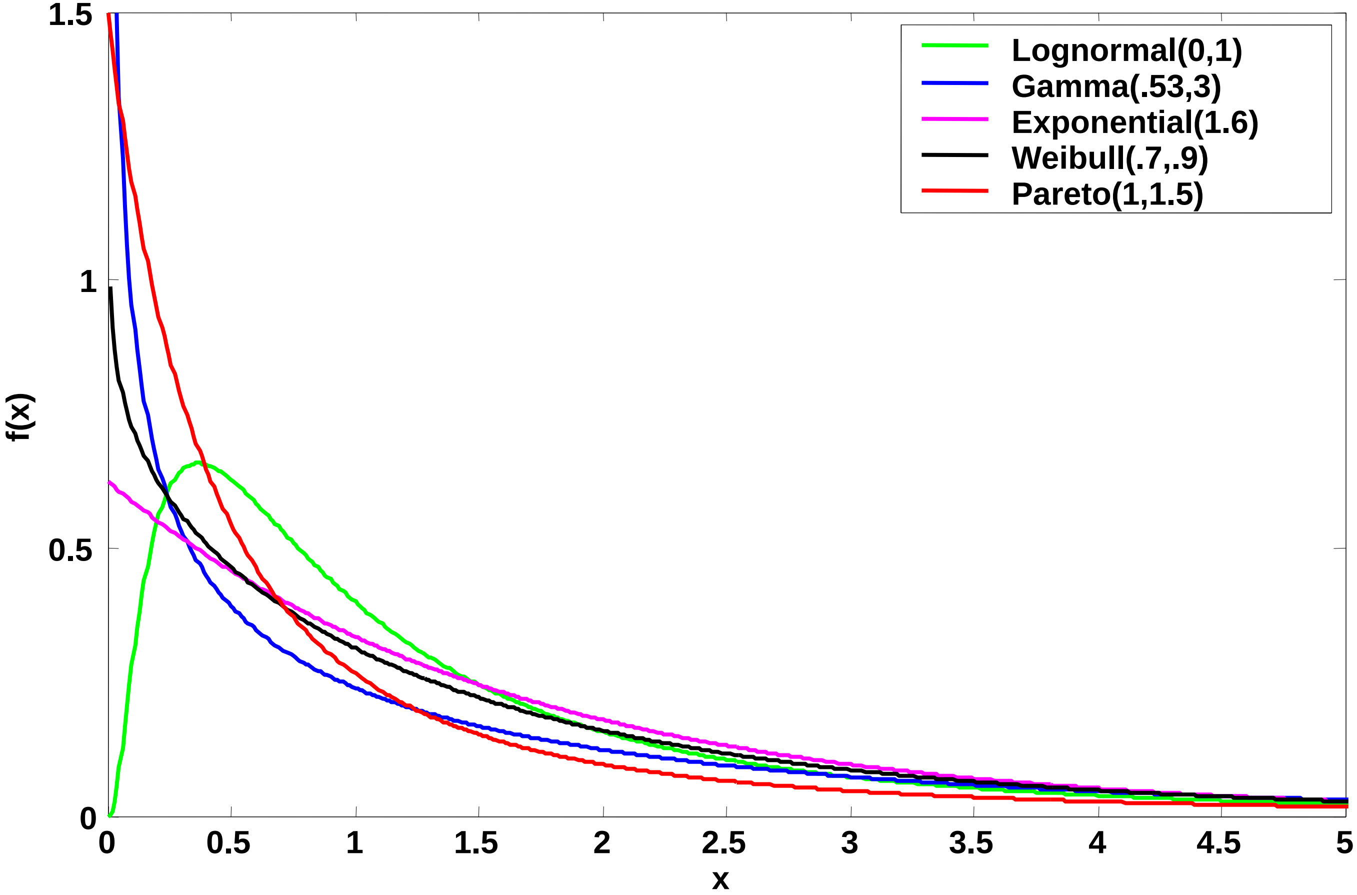
- Step 0: Datasets
  - One or more sets of comparable measurements
- Step 1: Model Selection (distribution)
  - Choose parametric family of models/distributions
- Step 2: Parameter Estimation
  - Take a strictly static view of data
  - Assume moment estimates exist/converge
- Step 3: Model Validation
  - Select "best-fitting" model
  - Rely on some "goodness-of-fit" criteria/metrics
  - Rely on some performance comparison

"Black box-type" analysis, "data-fitting" exercise

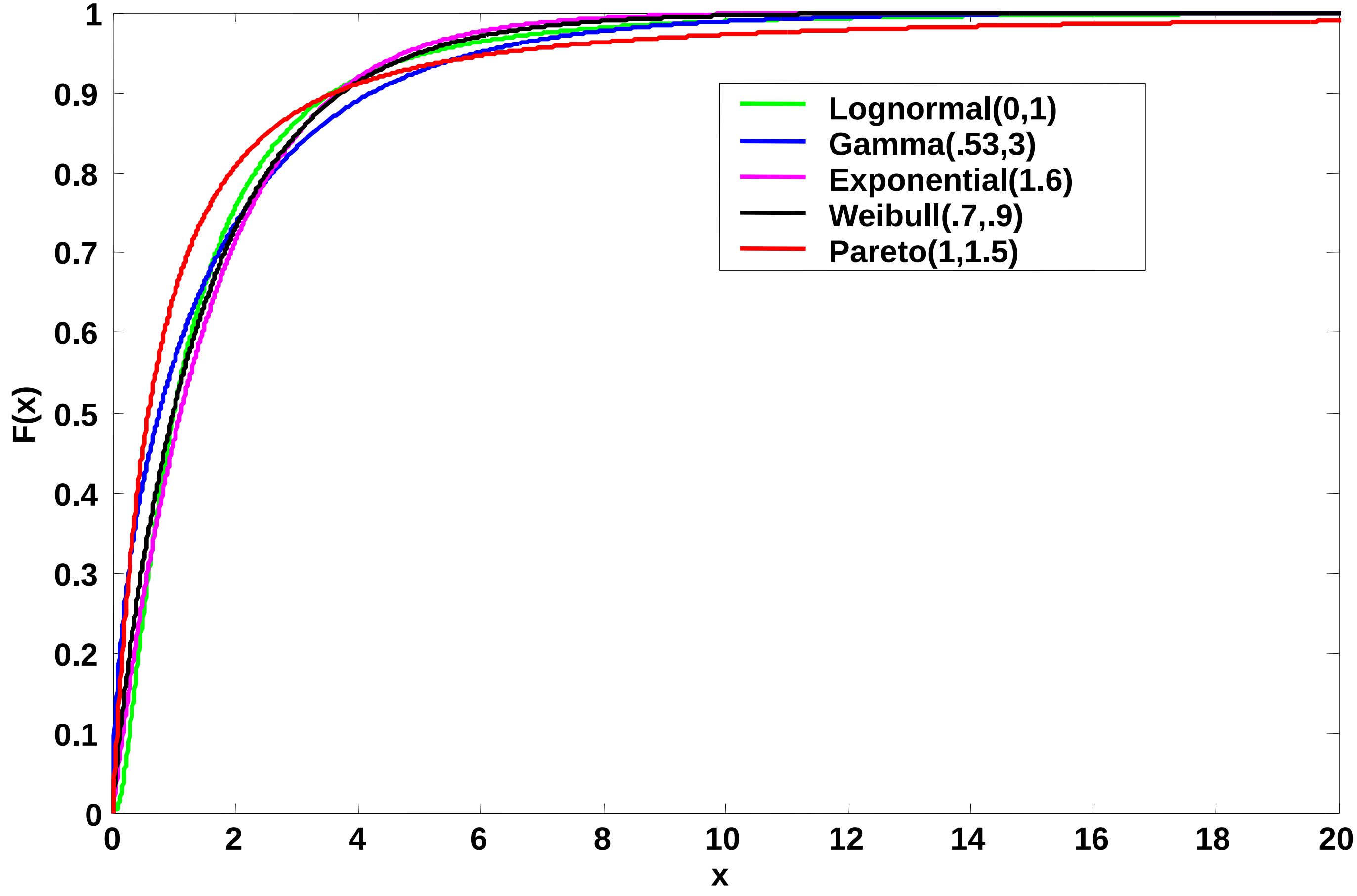
Some Illustrative Examples

- Some commonly-used plotting techniques
  - Probability density functions (pdf)
  - Cumulative distribution functions (CDF)
  - Complementary CDF (CCDF)
- Different plots emphasize different features
  - Main body of the distribution vs. tail
  - Variability vs. concentration
  - Uni- vs. multi-modal

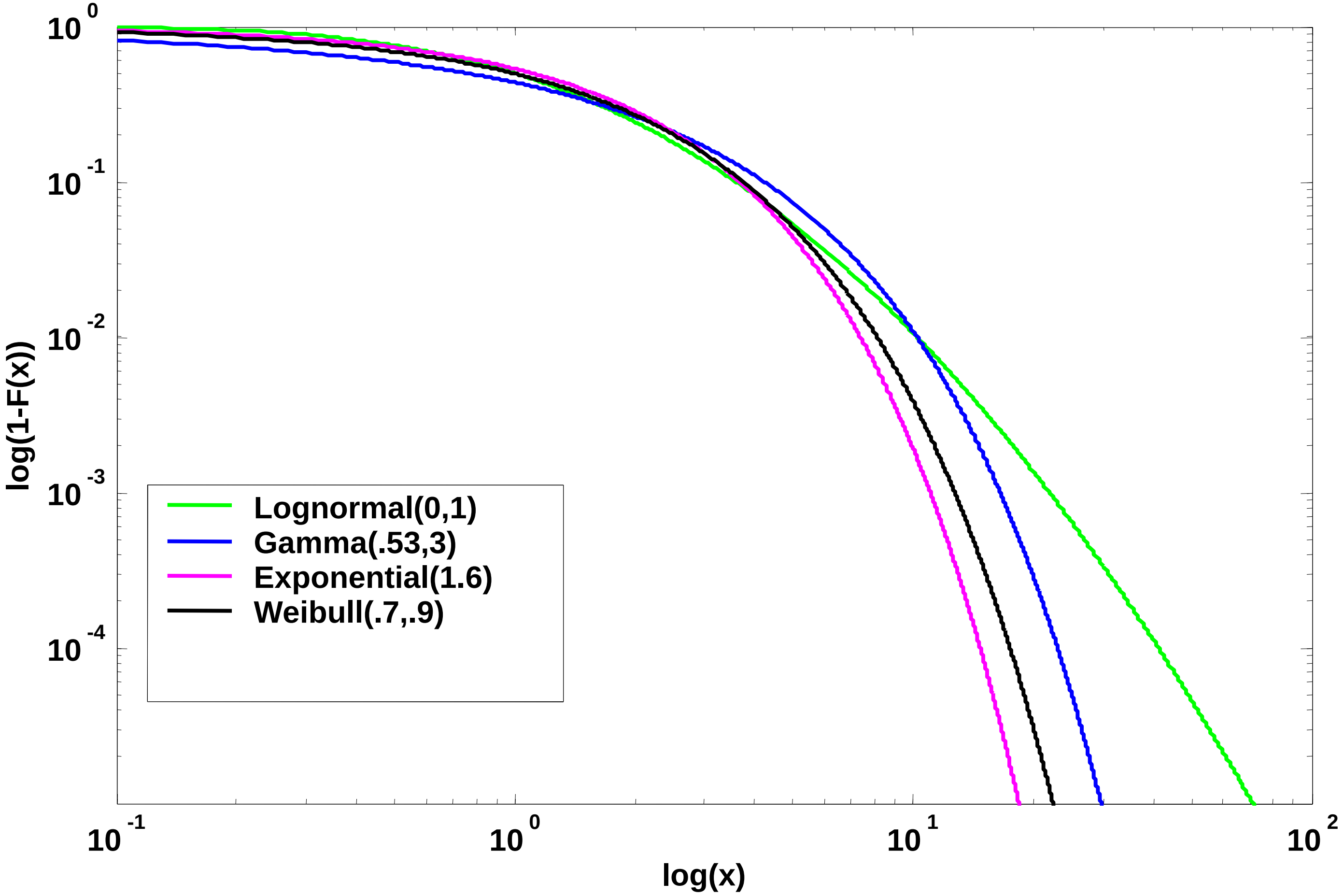
Probability density functions



Cumulative Distribution Function



Complementary CDFs



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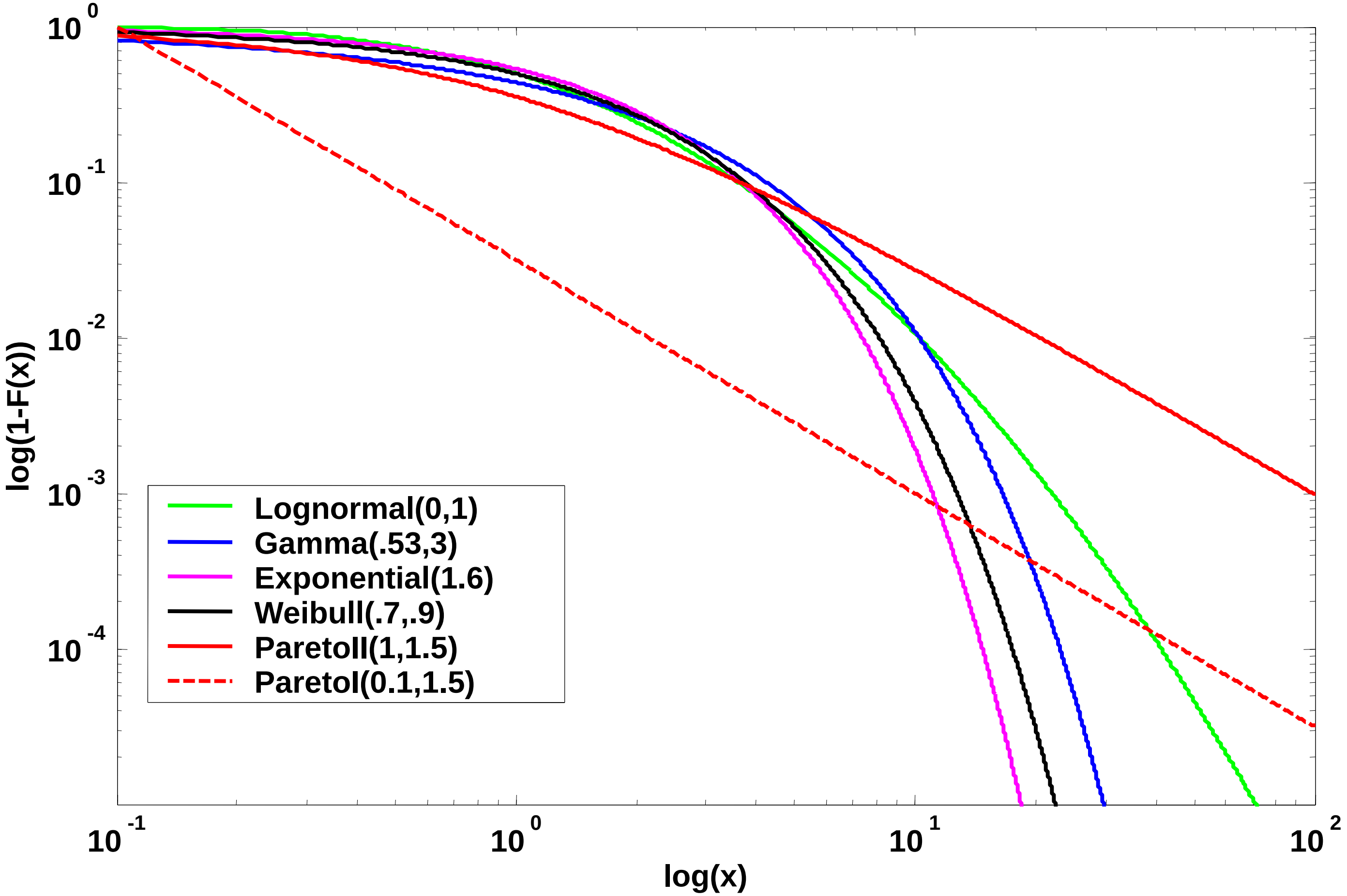


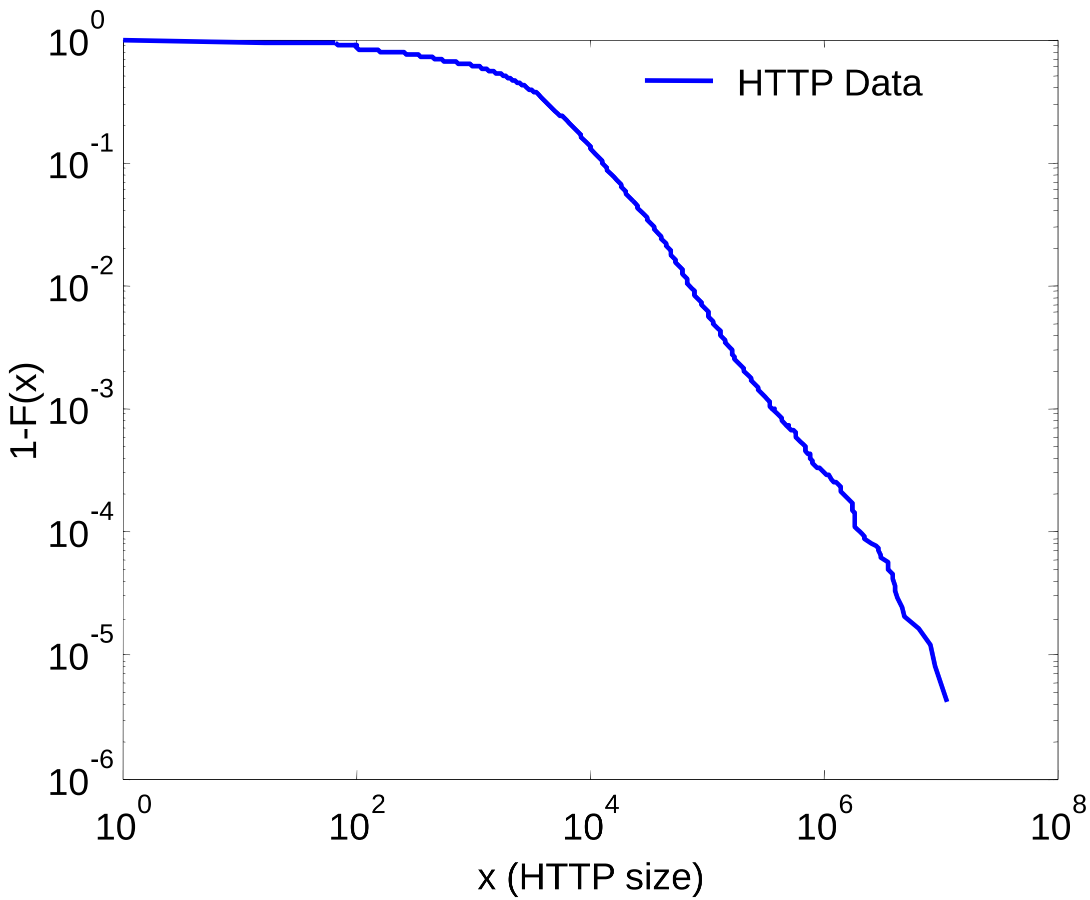
By Example

Internet Traffic

- HTTP Connection Sizes from 1996
- IP Flow Sizes (2001)

HTTP Connection Sizes (1996)

- 1 day of LBL's WAN traffic (in- and outbound)
- About 250,000 HTTP connection sizes (bytes)
- Courtesy of Vern Paxson



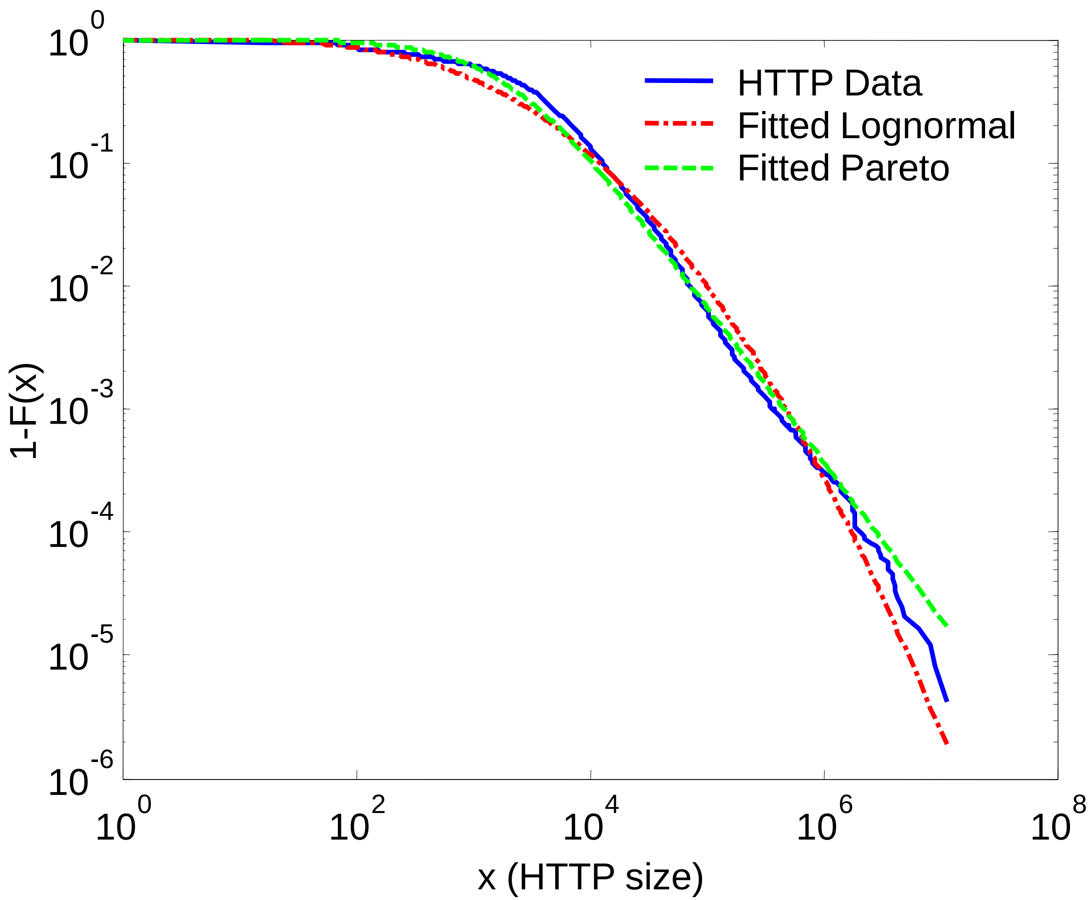
HTTP Connection Sizes (1996)

How to deal with "high variability"?

- Option 1: High variability = large, but finite variance
- Option 2: High variability = infinite variance

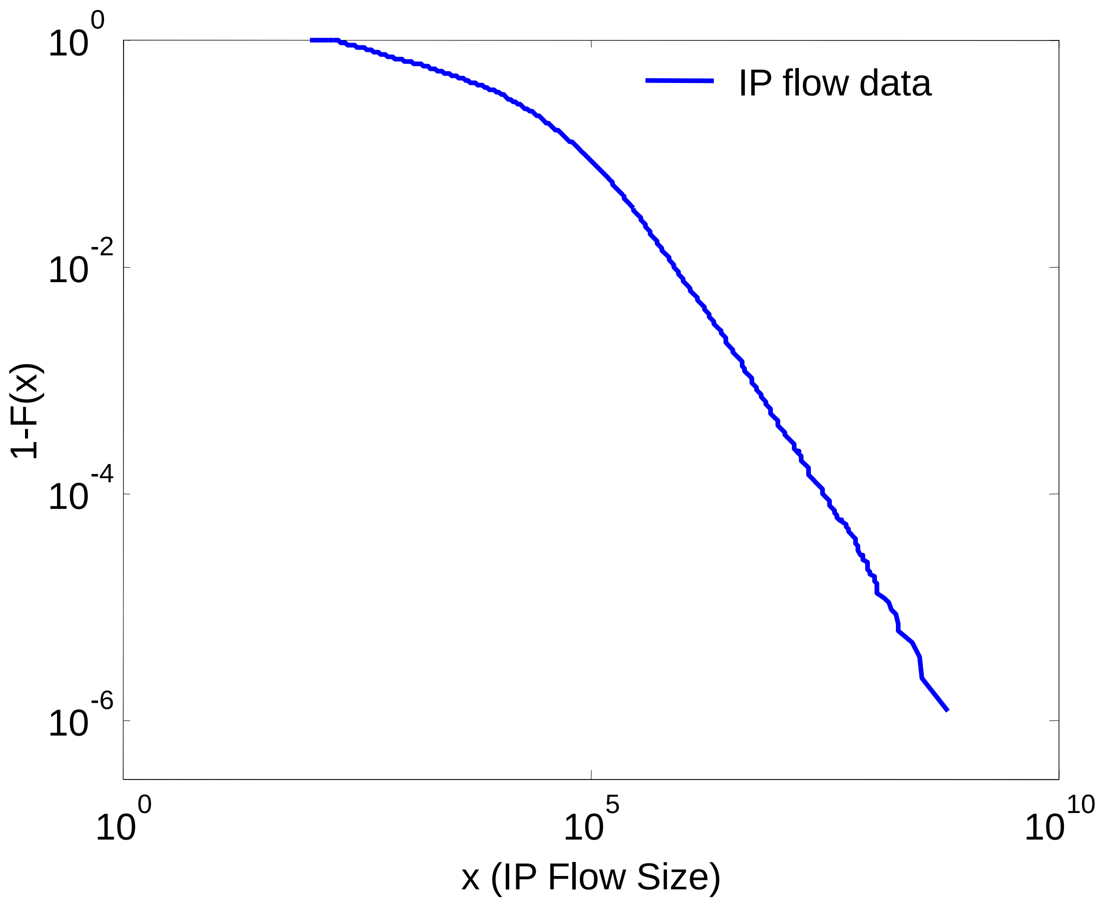
Fitted 2-parameter Lognormal (μ=6.75, σ=2.5)

Fitted 2-parameter Pareto (α=1.27, m=2000)



IP Flow Sizes (2001)

- 4-day period of traffic at Auckland
- About 800,000 IP flow sizes (bytes)
- Courtesy of NLANR and Joel Summers



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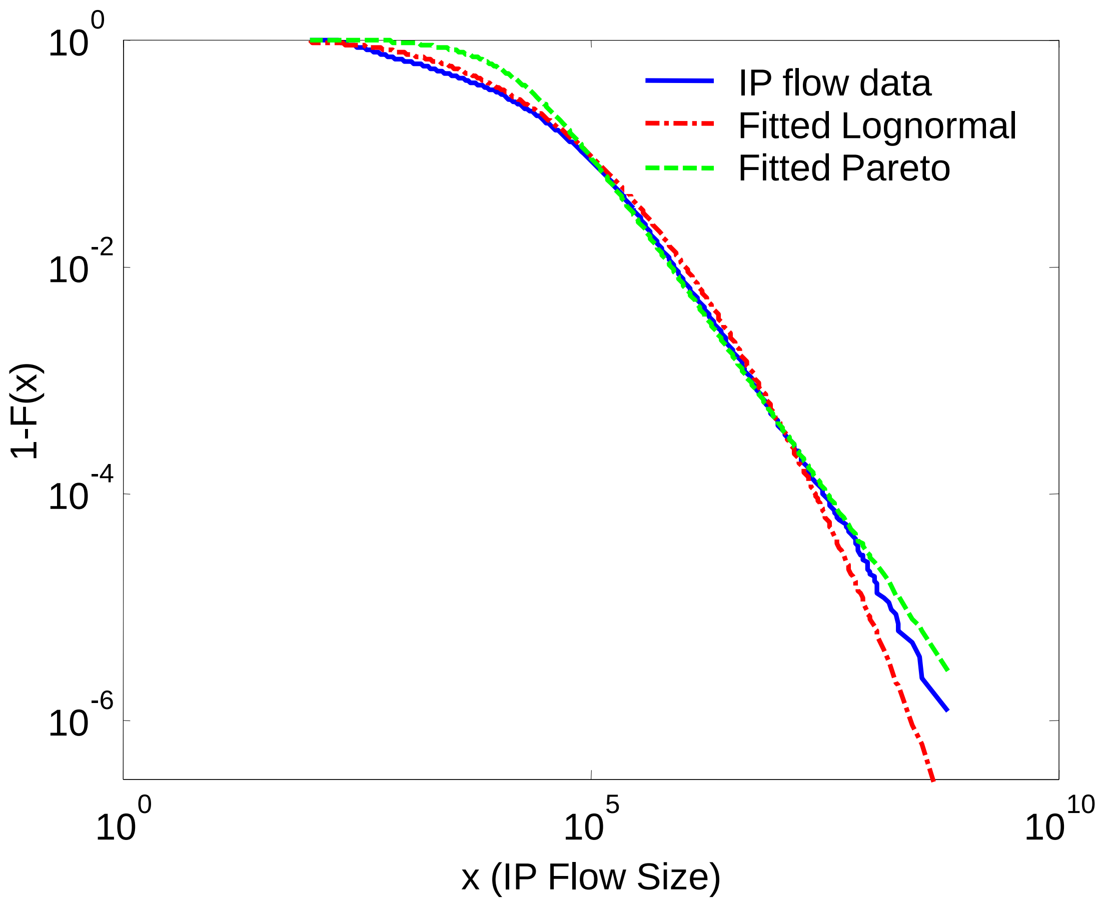
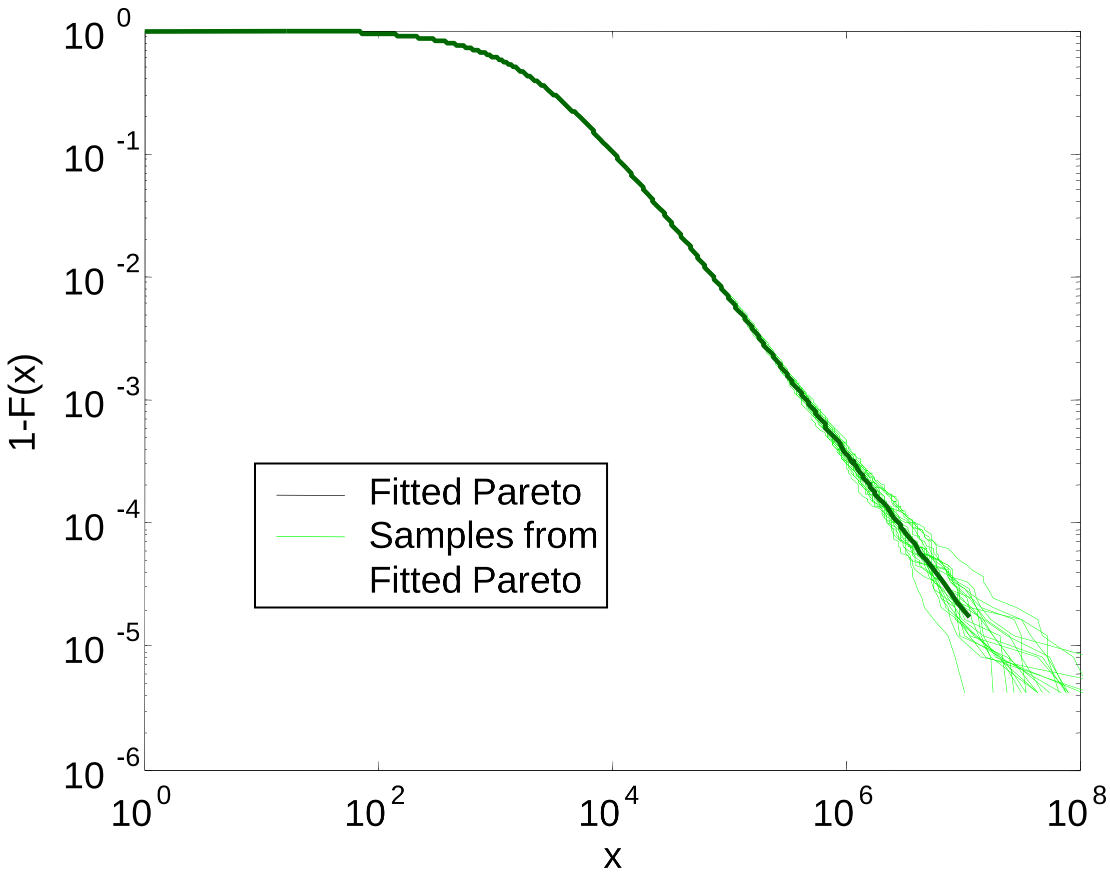
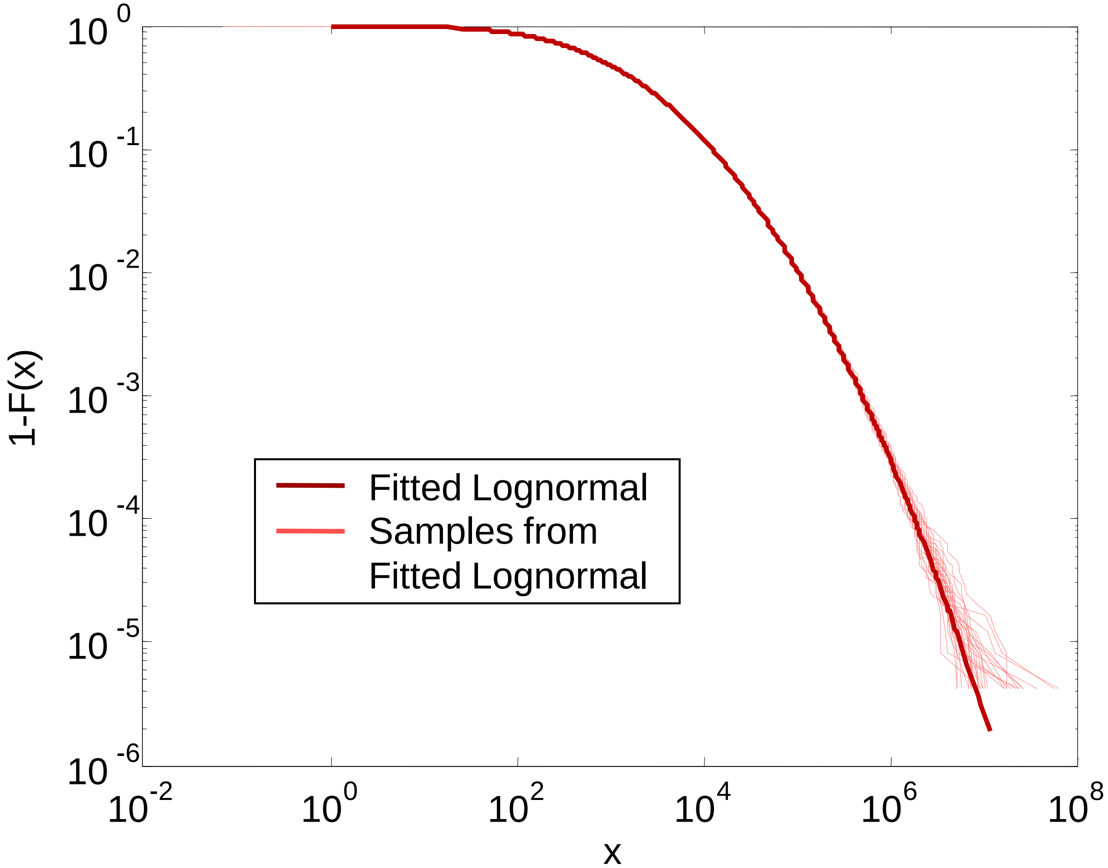
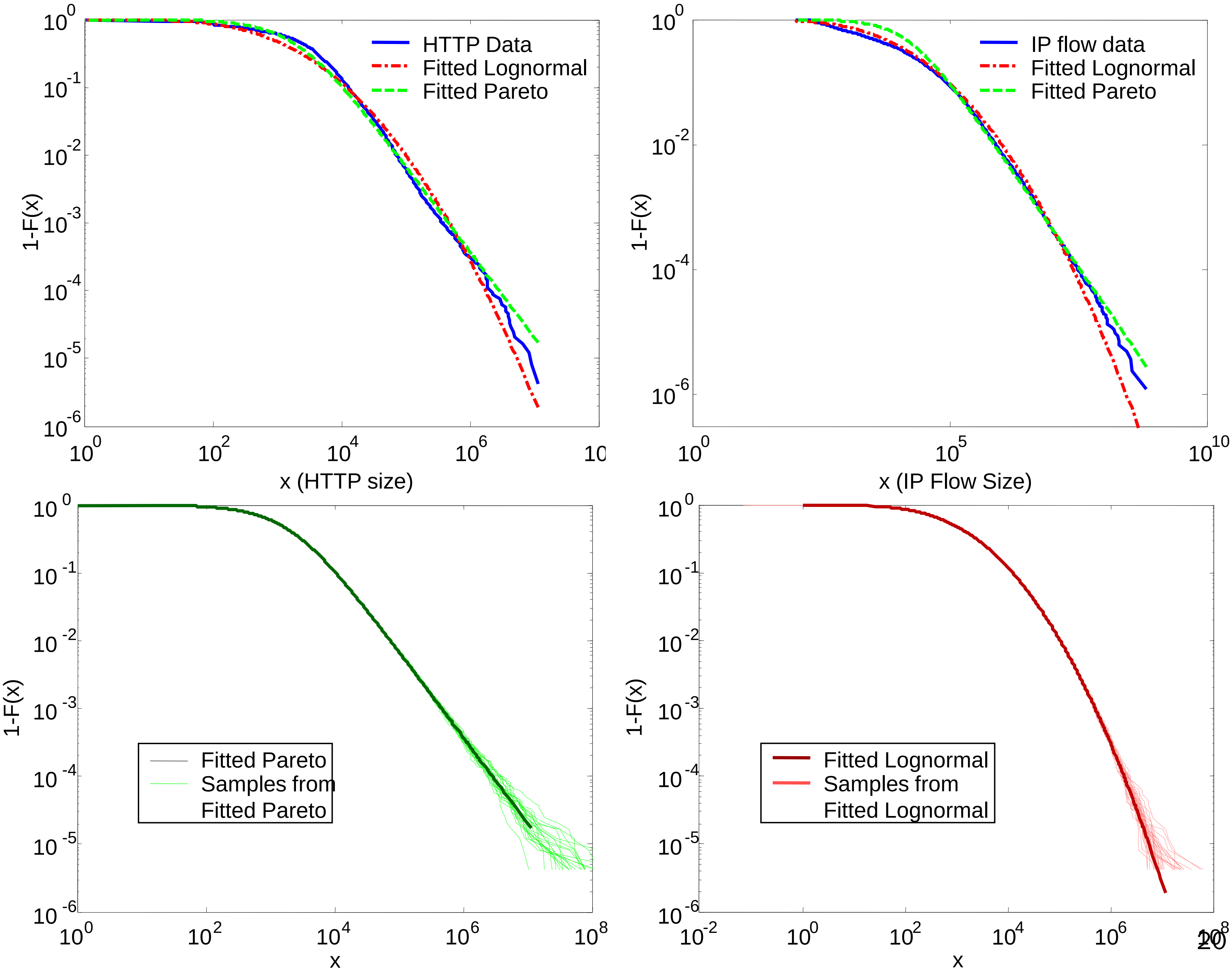


Samples from Pareto Distribution



Samples from Lognormal Distribution




HTTP Data
Fitted Lognormal
Fitted Pareto
1-F(x)
x (HTTP size)
IP flow data
Fitted Lognormal
Fitted Pareto
1-F(x)
x (IP Flow Size)
Fitted Pareto
Samples from
Fitted Pareto
1-F(x)
x
Fitted Lognormal
Samples from
Fitted Lognormal
1-F(x)
x

Traditional Modeling Approach

- Step 0: Datasets
- Step 1: Model Selection
- Step 2: Parameter Estimation
- Step 3: Model Validation

Criticism of Traditional Approach

- Highly predictable outcome
  - Always doable, no surprises
  - Cause for endless discussions (Downey'01)
- Curve fitting or data-fitting exercise
  - "more" always means "better" ...
  - Adding parameters improves fit
- Inadequate "goodness-of-fit" criteria due to
  - Voluminous data sets
  - Dependencies, high-variability, non-stationarities

Beyond Traditional Internet Modeling

- Requirement 1: Internal Model Consistency
  - Exploit high volume of available data
  - Learn from Mandelbrot and Tukey
  - Example: HTTP and IP traffic data

- Requirement 2: External Model Consistency
  - Exploit rich semantic of available data
  - Learn more from Mandelbrot and Cox
  - Example: Internet traffic

- Requirement 3: Resilience to Ambiguous Data
  - High quality vs low quality
  - High variability vs low variability
  - Example: Internet connectivity

Internal Model Consistency

- Reflects the fact that the size of any given dataset could really be anything (why 1 hour? 1 day? 10 min?)

- Take dynamic view of data
  - Rely on traditional modeling approach for initial (small) subset of available data (model M(0))
  - Consider successively larger subsets (models M(k))
  - Analyze resulting family of models M(0),…,M(n)

- Approach: Tukey's "borrowing strength" idea
  - Borrowing strength from large data sets
  - Simple way to exploit high-volume data sets
  - Traditional modeling as a means, not as an end in itself

Internal Model Consistency (cont.)

- Internally consistent family of models
  - Parameter estimates converge quickly/robustly
  - 95% Confidence intervals become nested

- Internally inconsistent family of models
  - Parameter estimates don't converge
  - 95% CI's don't overlap
  - "Patchwork of fixes"

Illustration: Lognormal Family of Models for HTTP Data

- Lognormal model assumes finite variance
- Tool: Mandelbrot's "sequential moment plots"
  - Plot moment estimates as a function of n
  - Plot corresponding 95% CI as a function of n
  - Look for convergence/divergence as n approaches the full sample size
- Practical implementation
  - Working with raw data
  - Working with transformations of raw data
  - Working with random permutation of transformations of raw data

Sequential Moment Plots: Raw Data

- Let D be original data set of size N
- Build sequential models M_0, M_1,..., M_N using nested data sets: $D_0 \subset D_1 \subset \ldots \subset D$ of size $N_0 < N_1 < \ldots < N$
- Plot sample STD as a function of n (sample size)

x10

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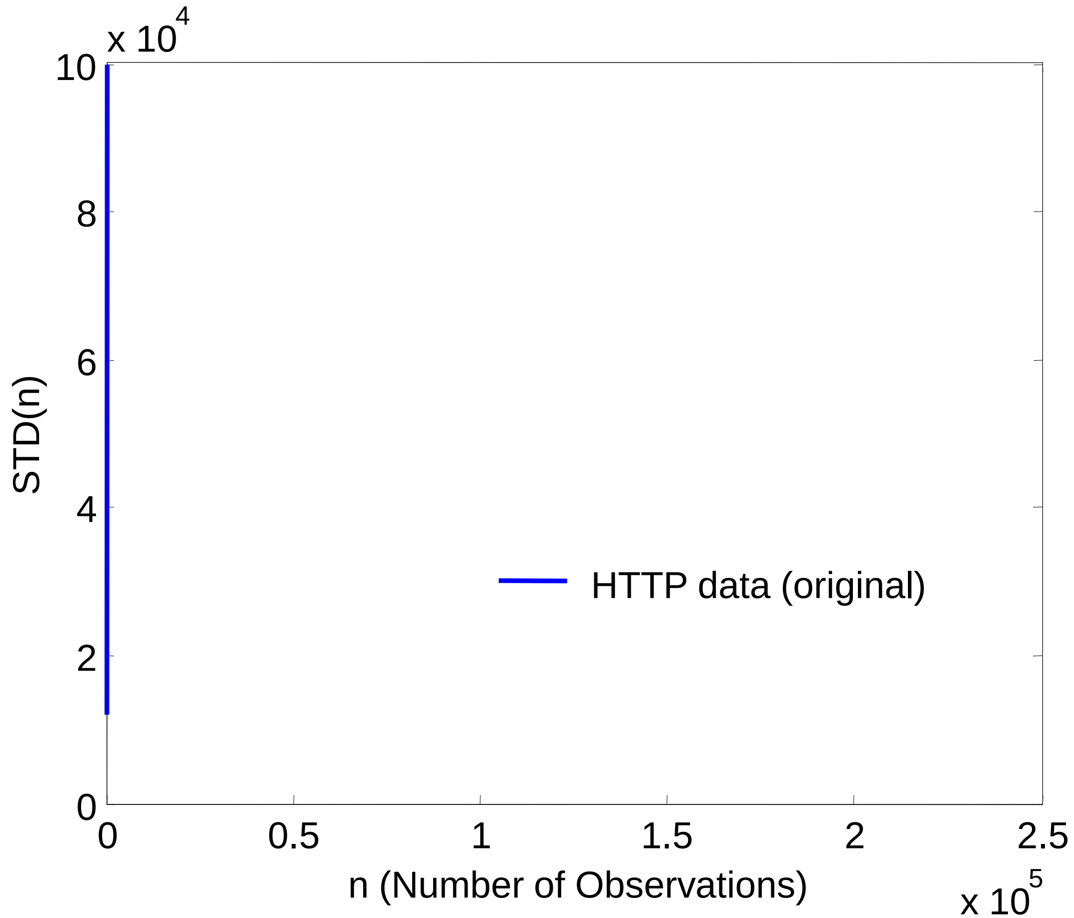

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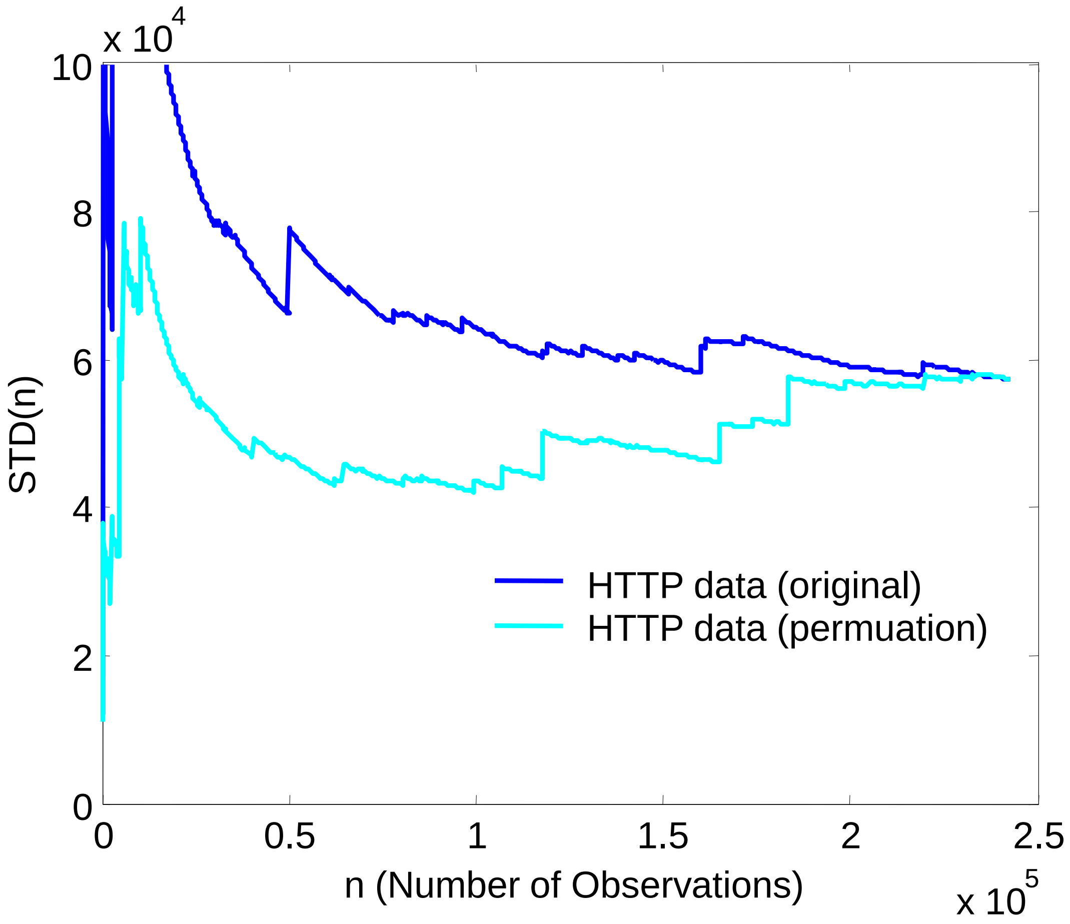

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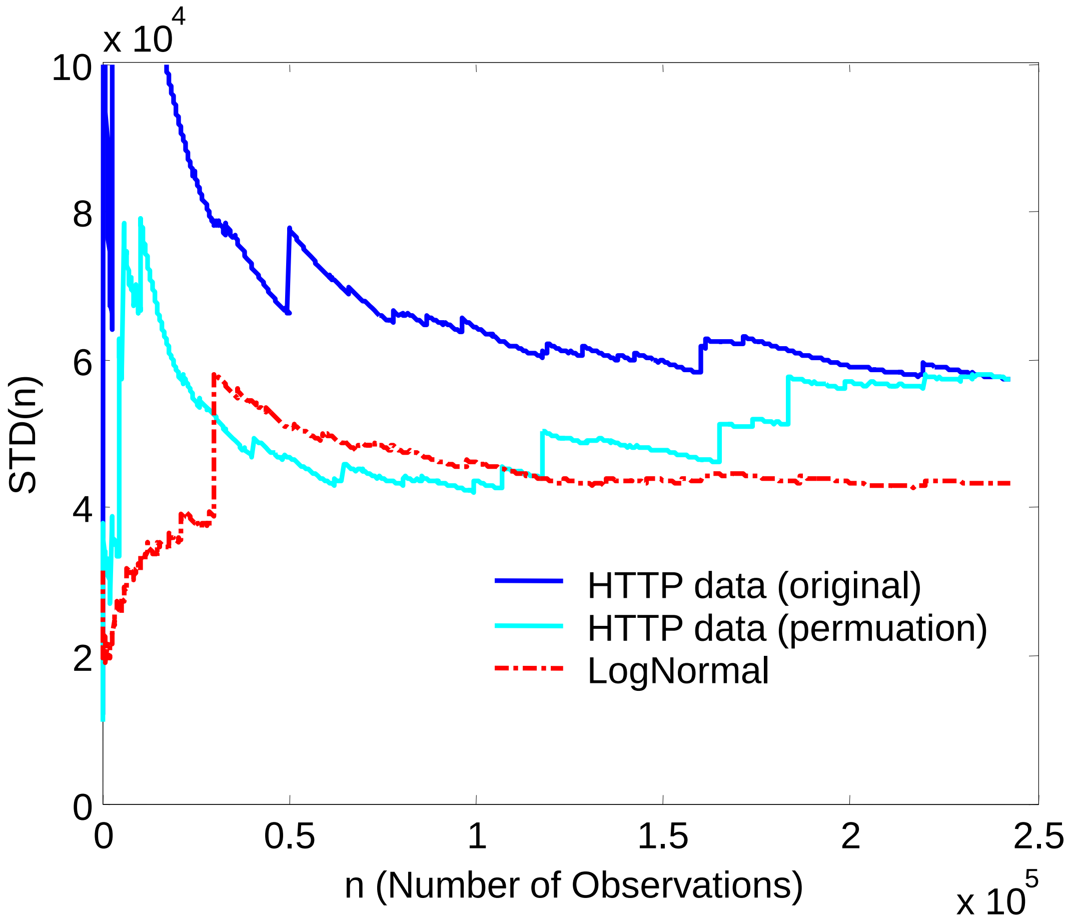

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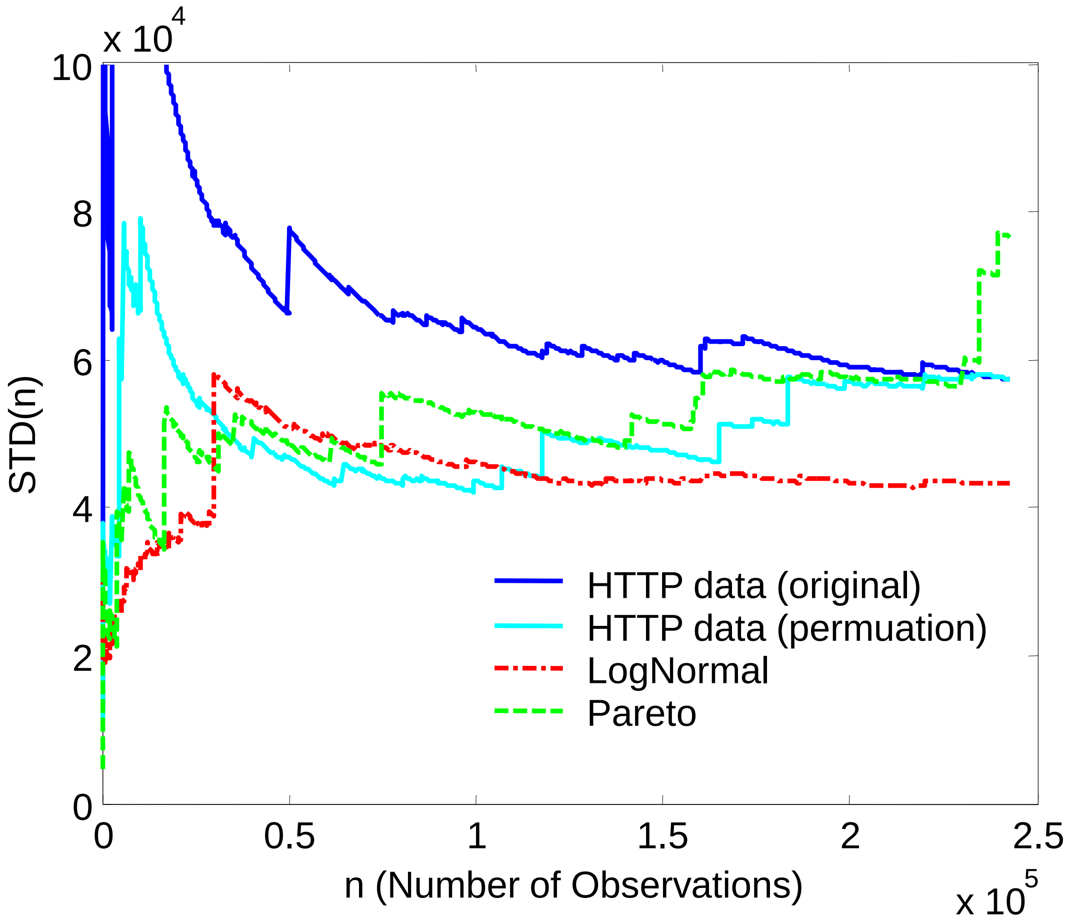

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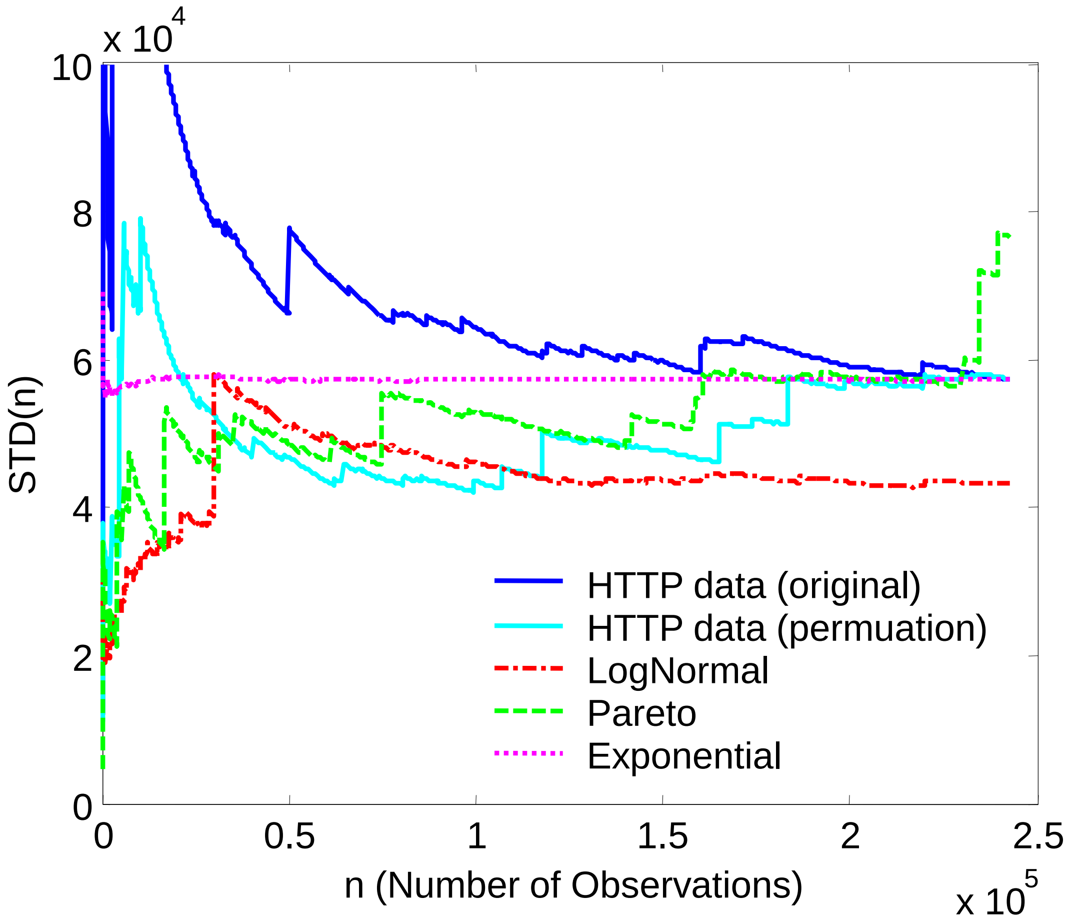


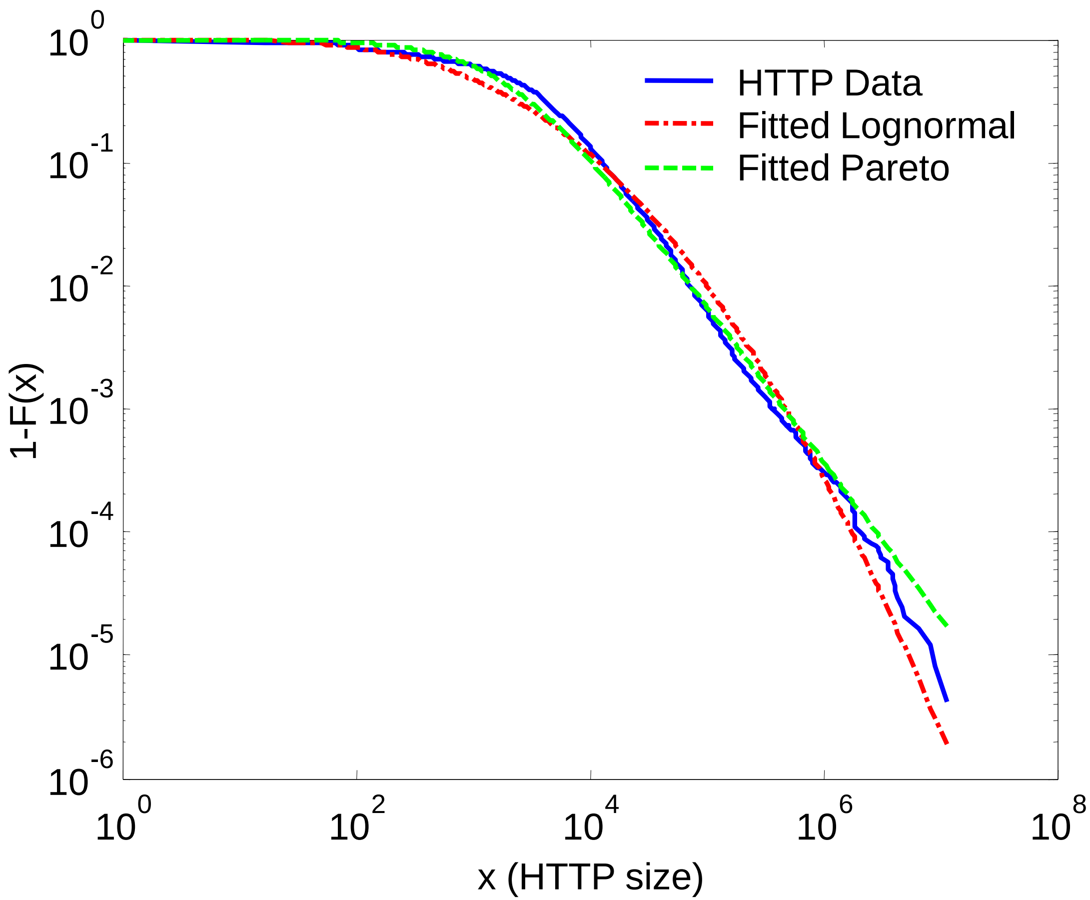
Sequential Parameter Estimates: Log-transformed Raw Data

Focus on the σ(n) parameter of the fitted lognormal model M(n)

- Behavior as its estimate as a function of n
- Behavior of corresponding 95% confidence intervals as a fct. of n

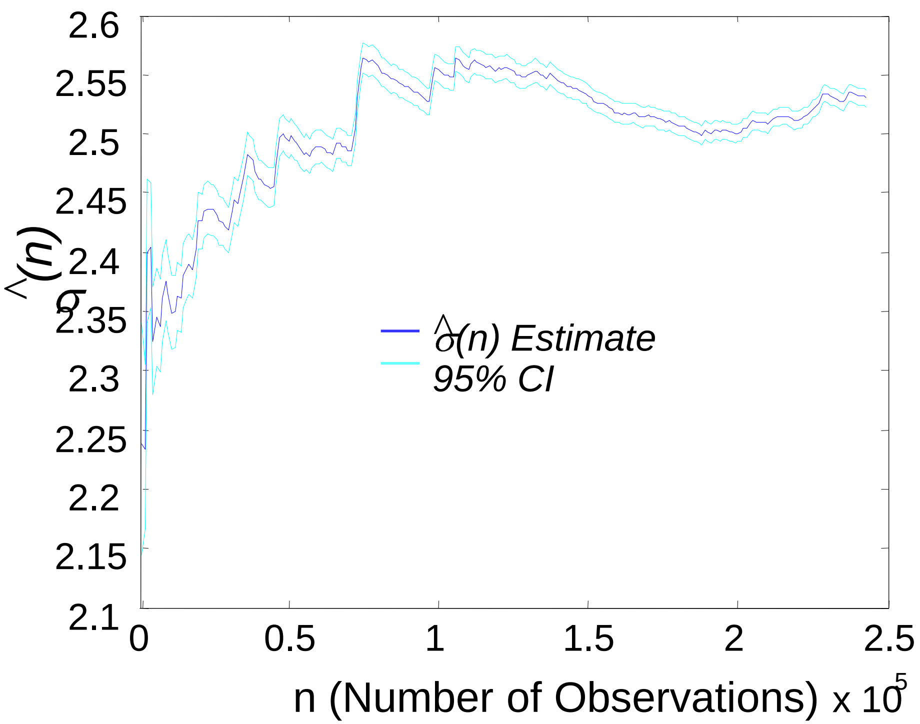
Fitted 2-parameter Lognormal (μ=6.75, σ=2.5)

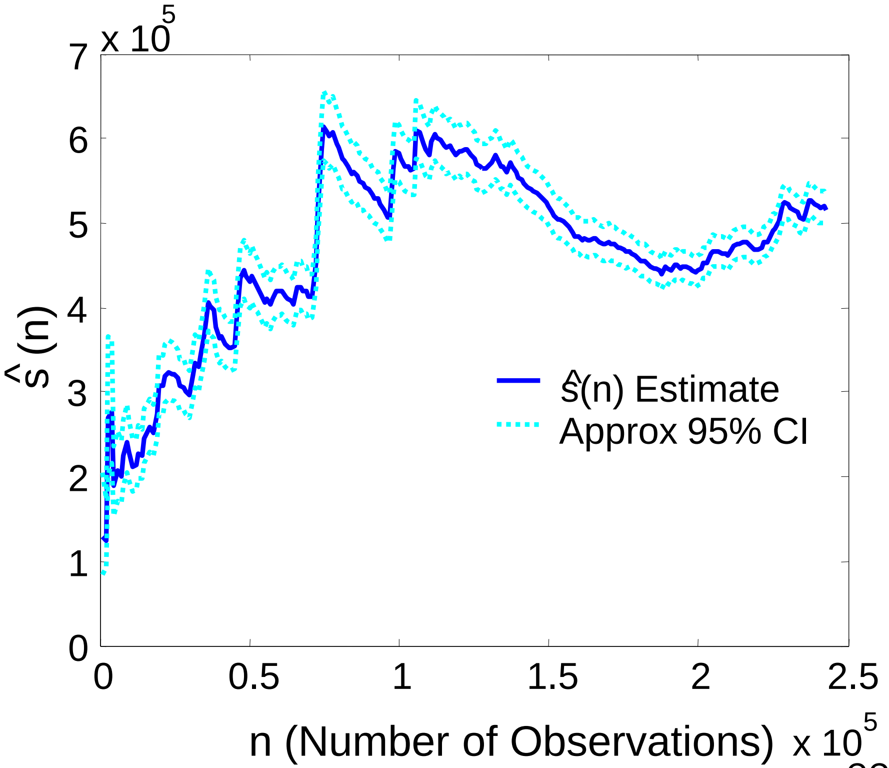
Fitted 2-parameter Pareto (α=1.27, m=2000)



Sequential Parameter Estimates: Log-transformed Raw Data

- Sequential estimates of parameter $\sigma(n)$ for fitted Lognormal model M_n, together with 95% CI





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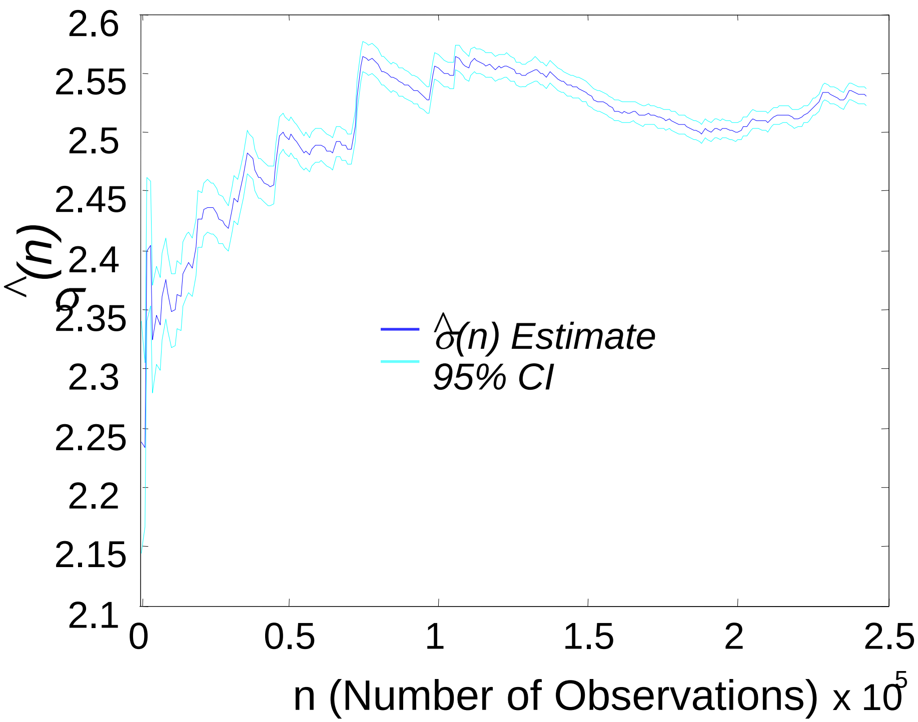
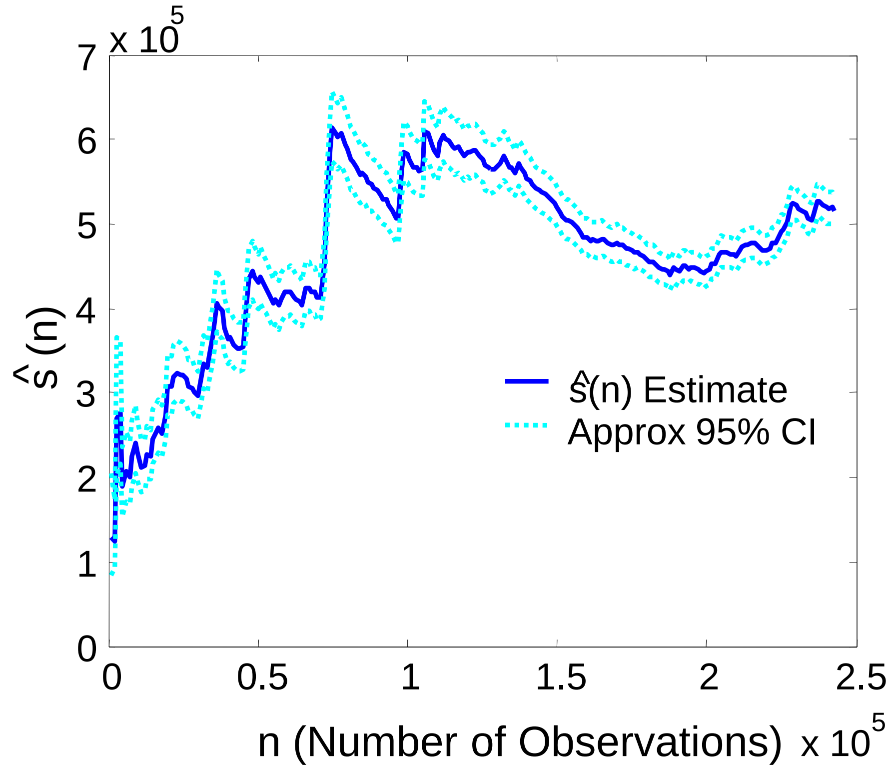
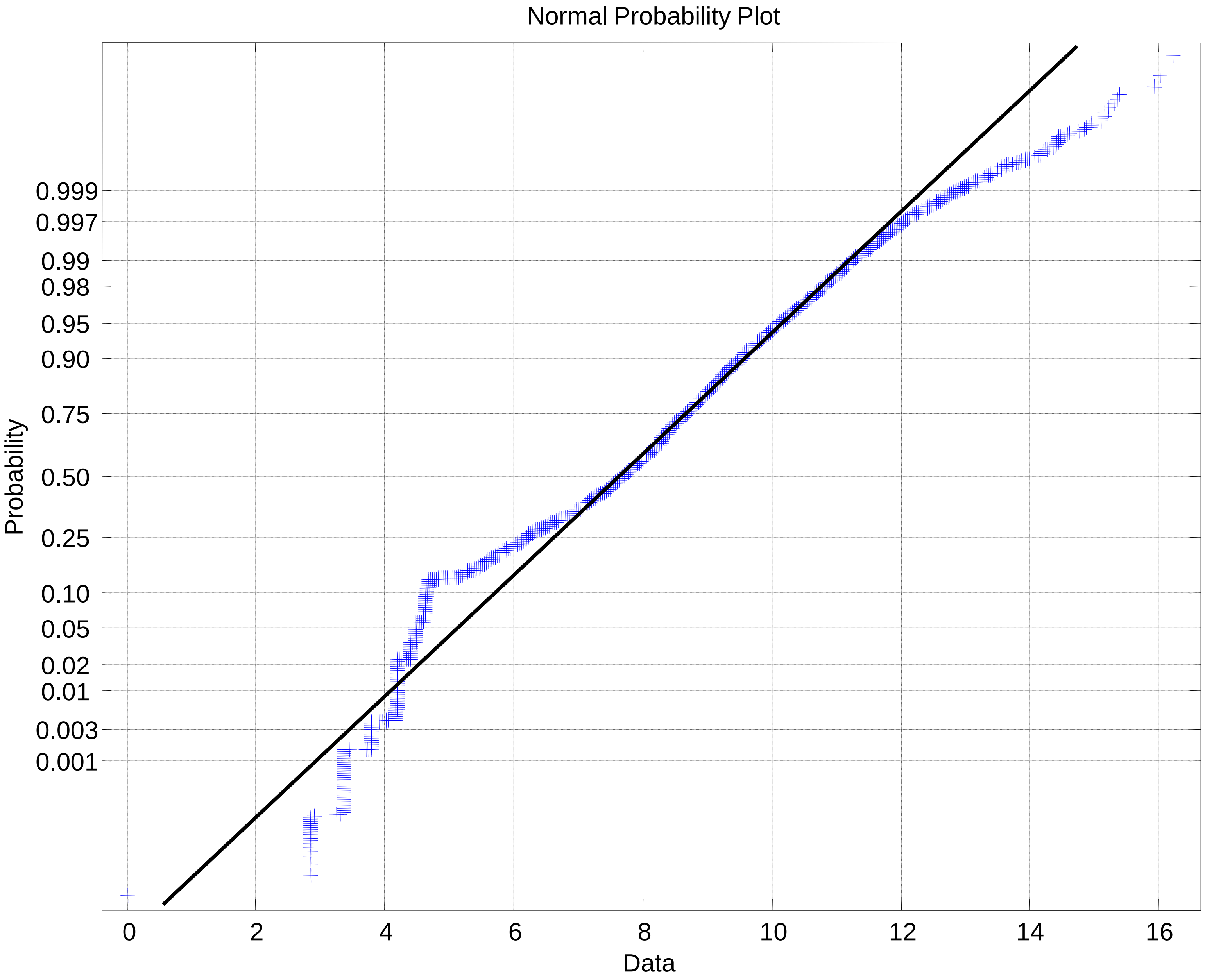




- Individual fitted lognormals appear adequate, but successive models are inconsistent (non-overlapping CIs)
- Minor differences in σ(n) estimate give very substantial differences for the standard deviation estimates s(n)

Does log-transformed data fit a normal?



The Case against the Lognormal Family of Models for HTTP Data

- The lognormal model assumes
  - existence/convergence of 2^{nd} moment
  - parameter estimates converge
- However, sequential moment plot indicates
  - non-existence/divergence of 2^{nd} moment
- However, sequential parameter estimate plot indicates
  - inherently inconsistent parameter estimates
- "Patchwork of fixes" (Mandelbrot)

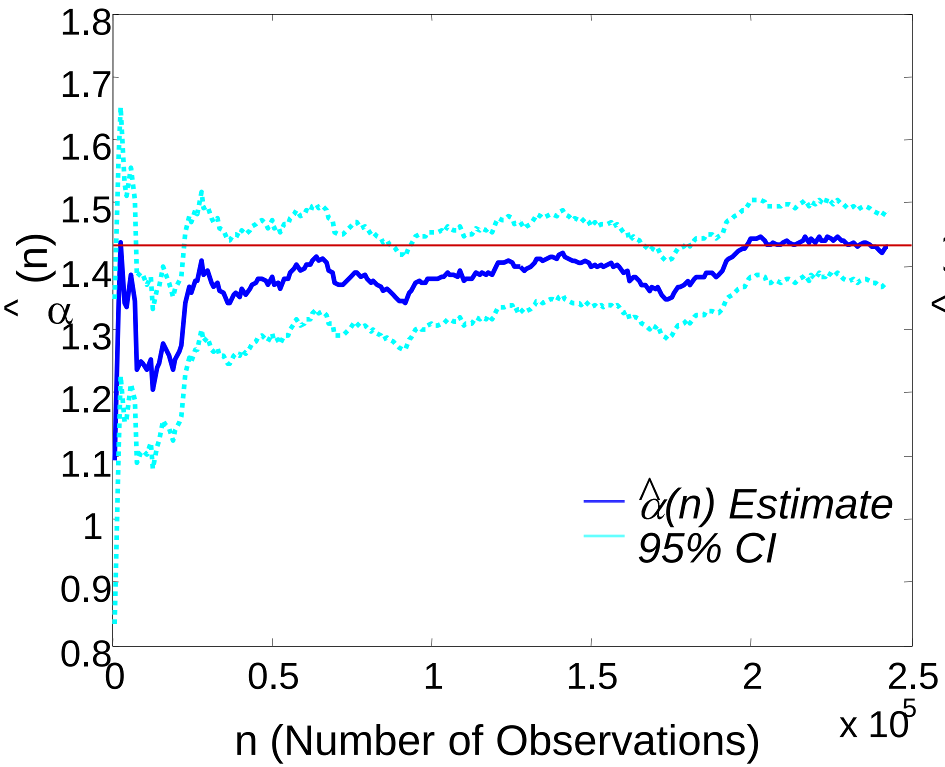
Example of being "certifiably wrong"

Illustration: Pareto Family of Models for HTTP Data

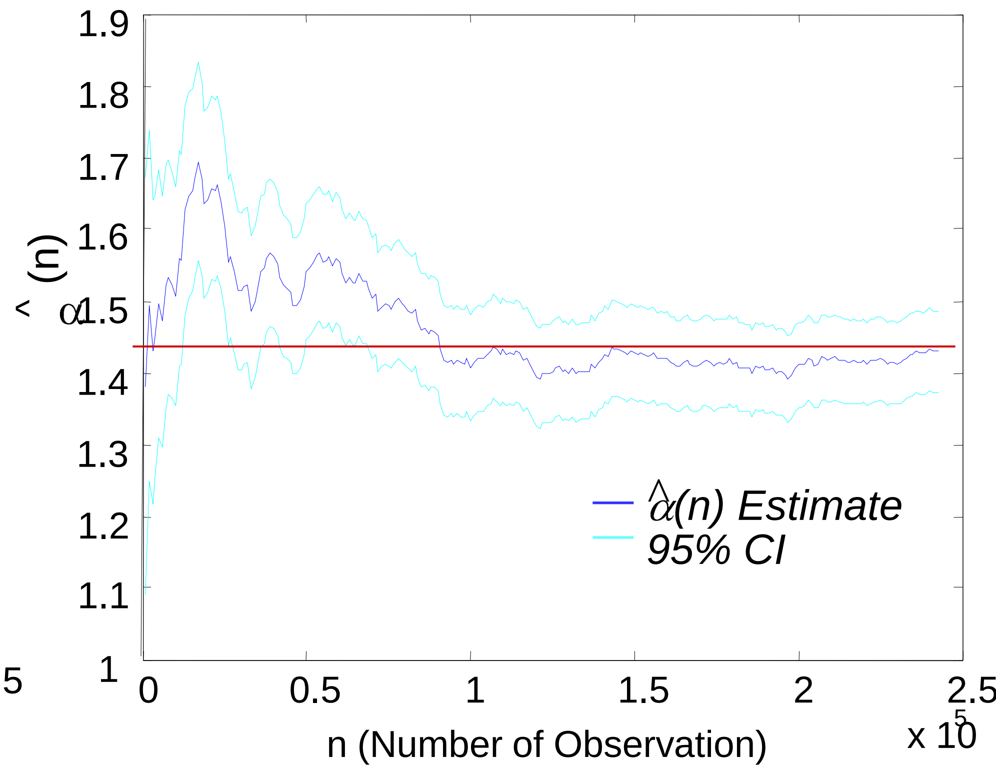
- Pareto model assumes infinite variance, but is defined in terms of tail index α
- Tool: "Sequential tail index estimate plots"
  - Plot tail index estimates as a function of n
  - Plot corresponding 95% CI as a function of n
  - Look for convergence/divergence as n approaches the full sample size
- Practical implementation
  - Working with raw data
  - Working with random permutation of raw data

HTTP: Sequential Tail Index Estimate Plots

- Sequential estimates of parameter $\alpha(n)$ for fitted Pareto model M_n, together with 95% CI

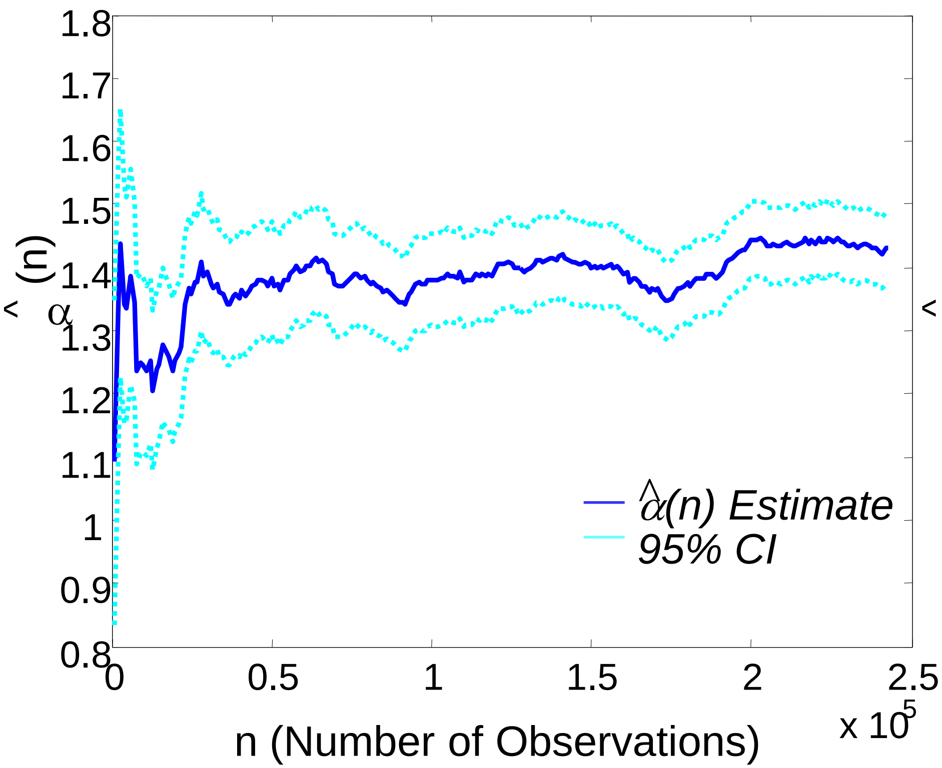


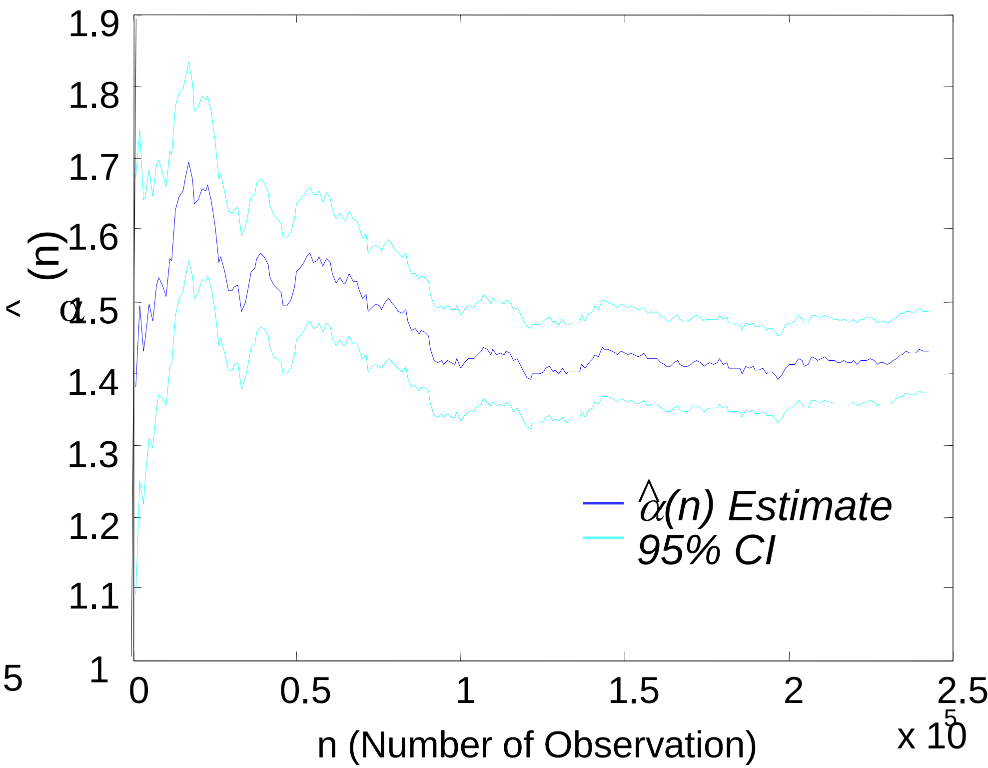
Raw Data



Random permutation of raw data

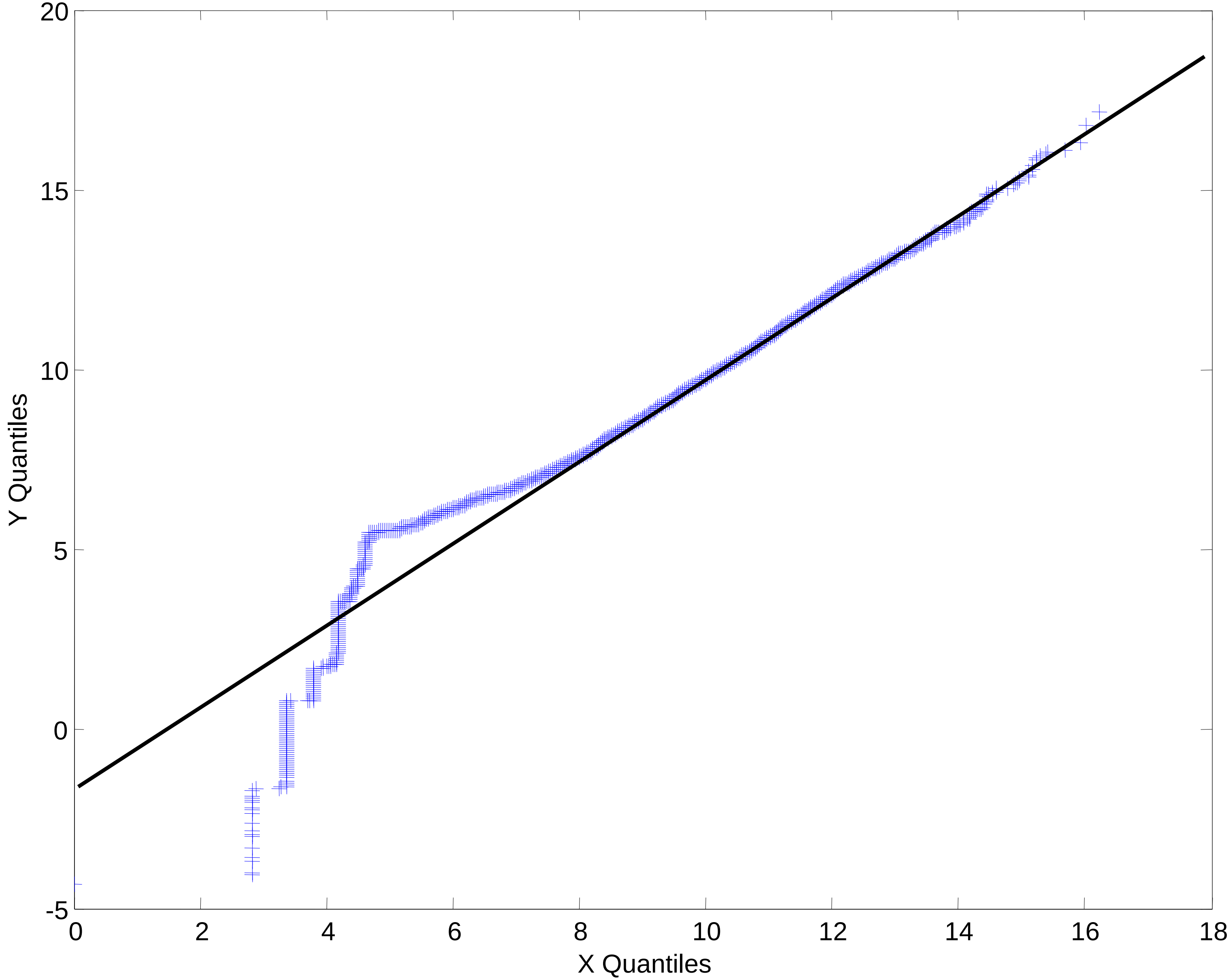
HTTP: Sequential Tail Index Estimate Plots





- Successive fitted Paretos appear largely consistent with one another (i.e. overlapping CIs)

HTTP: Does the data fit a Pareto?



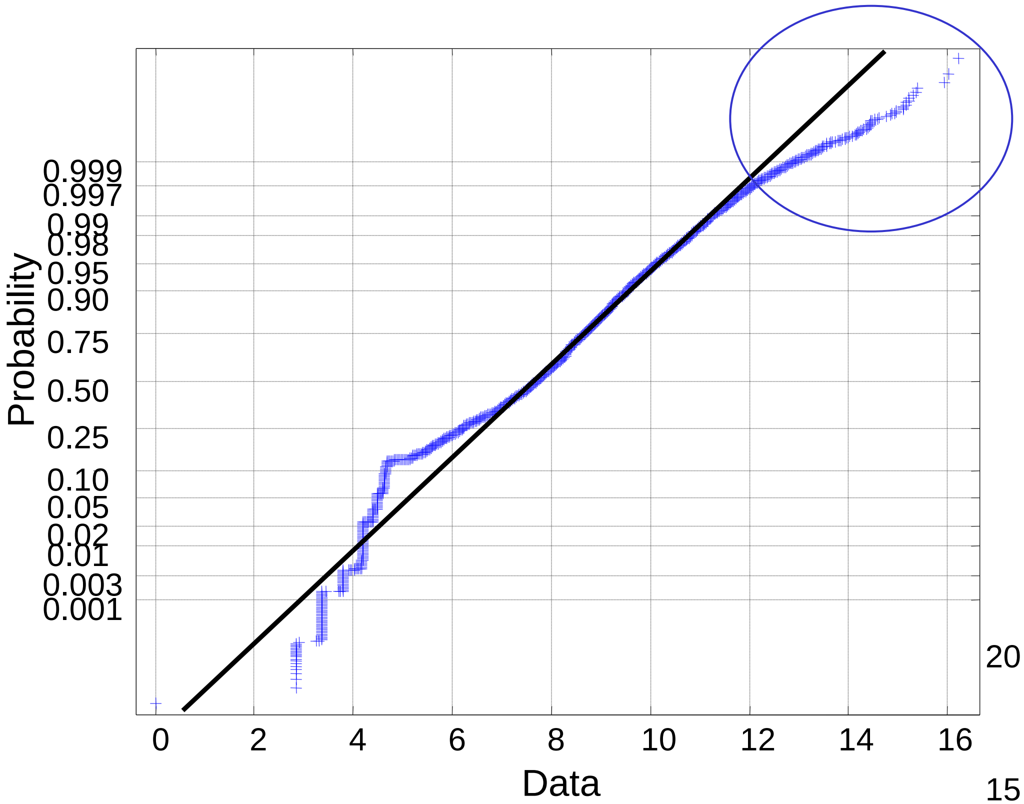
The Case for the Pareto Family of Models for HTTP Data

- The Pareto model assumes
  - 2^{nd} moment estimates are infinite/diverge
  - Tail index estimates converge
- Sequential moment plot indicates
  - 2^{nd} moments are infinite/diverge
- Sequential parameter estimate plot indicates
  - Inherently consistent estimates

- The "creativity" of power-law distributions
  - In theory: Infinite moments
  - In practice: Divergent sequential moment plots

- Scientifically "economical" modeling
  - When more data doesn't mean more parameters
  - More data simply means more accurate estimates
  - "Parsimonious" modeling
  - Trading "goodness-of-fit" for "robustness"

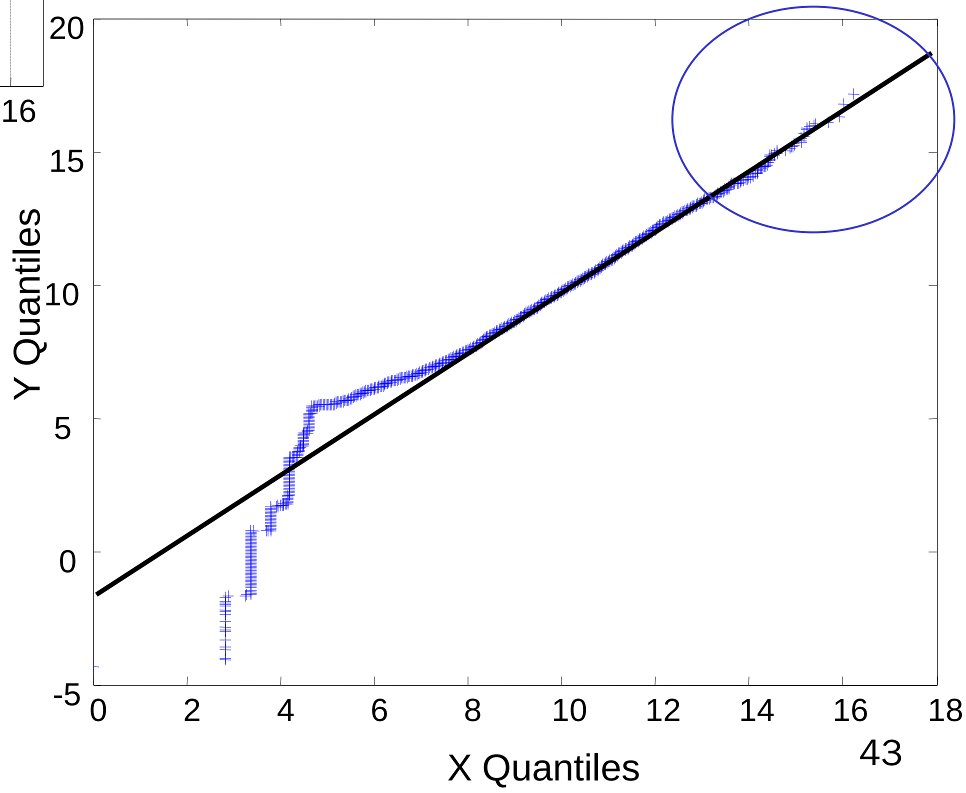
Example of being "approximately right"



“All models are wrong… but some are less so …” (P.G.E. Box)

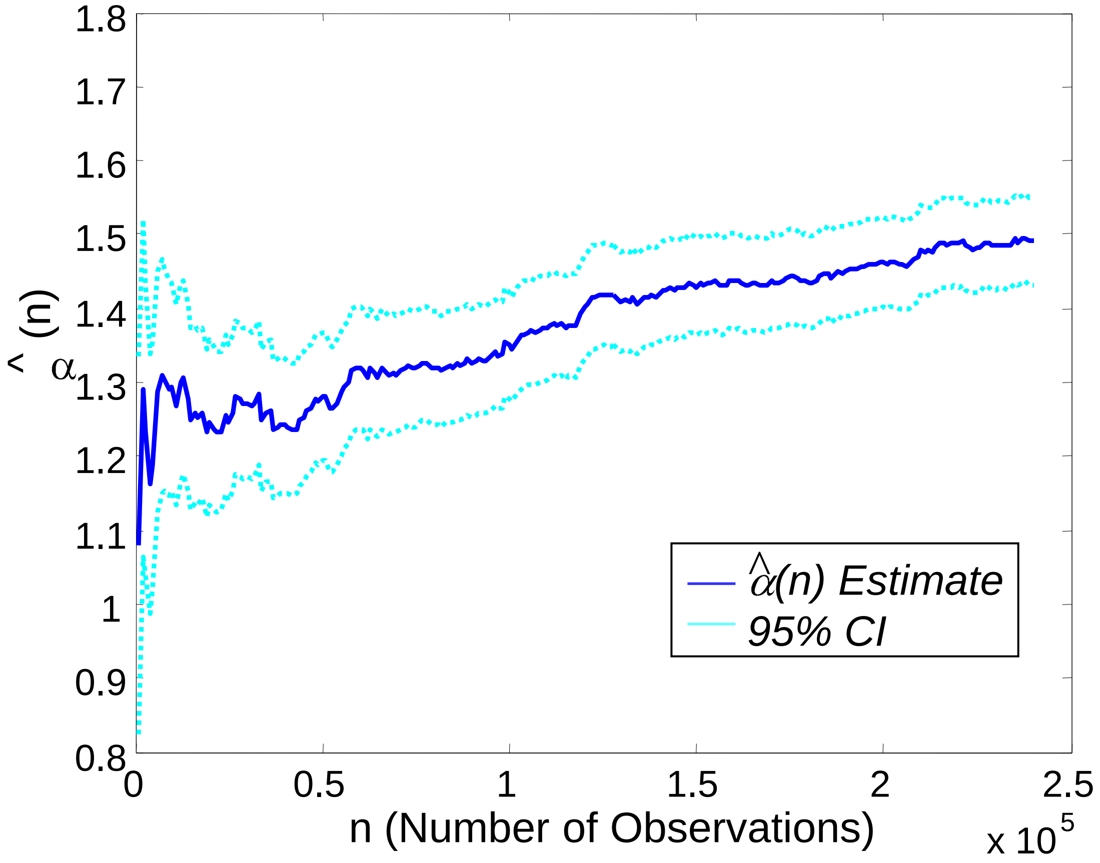
HTTP: Fitted Lognormal

HTTP: Fitted Pareto



Some Sanity Checks

- Fitting Pareto model to Lognormal sample
  - Generate iid sample from a Lognormal model
  - Check sequential tail index estimate plot



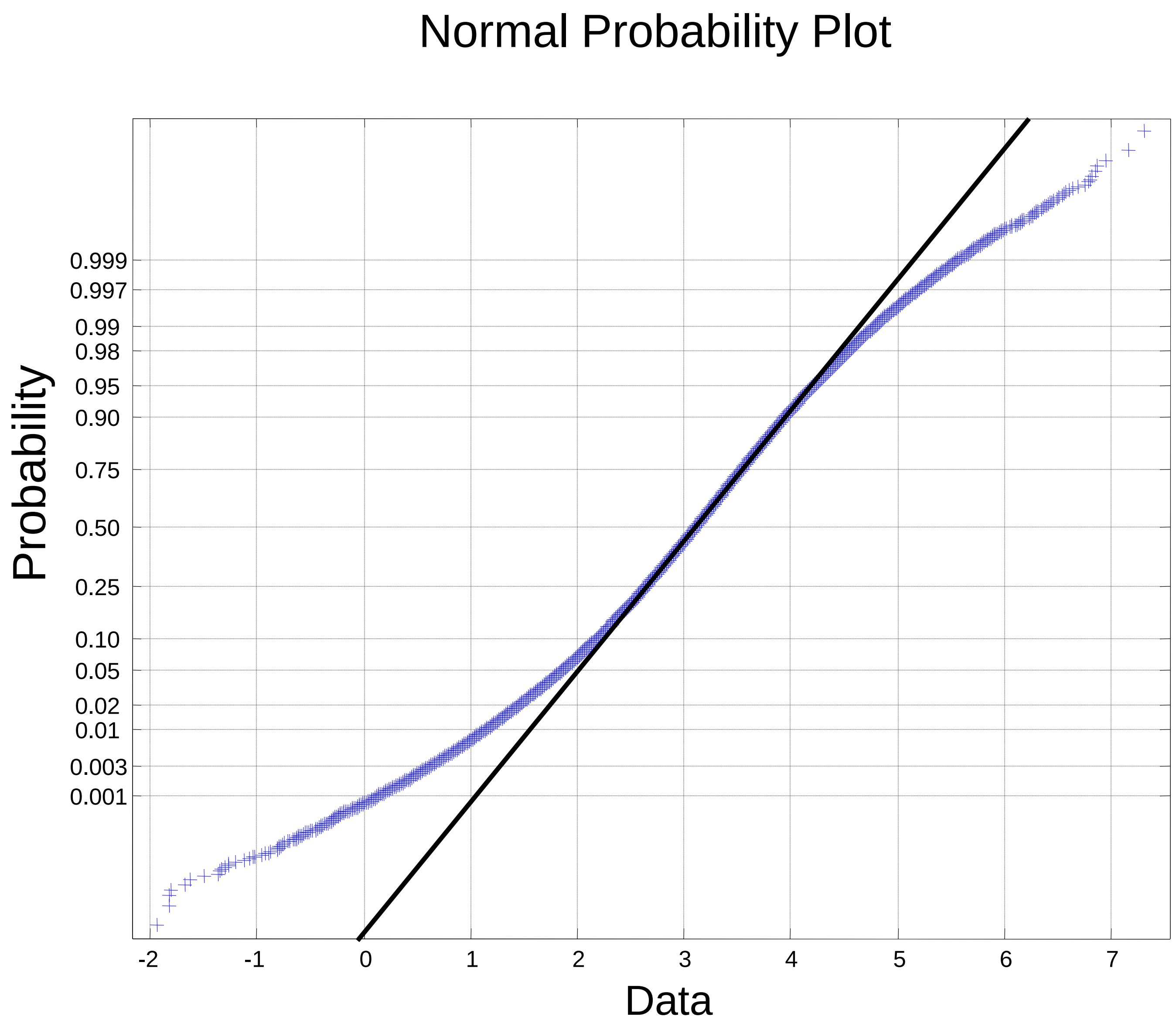
Using a Pareto model for lognormal data

Some Sanity Checks

- Fitting Pareto model to Lognormal sample
  - Generate iid sample from a Lognormal model
  - Check sequential tail index estimate plot

- Result: sequential tail index estimates diverge

- Fitting Lognormal model to Pareto sample
  - Generate iid sample from a Pareto model
  - Check normal probability plot



Using a lognormal model for Pareto data

Some Sanity Checks

- Fitting Pareto model to Lognormal sample
  - Generate iid sample from a Lognormal model
  - Check sequential tail index estimate plot
- Result: sequential tail index estimates diverge

- Fitting Lognormal model to Pareto sample
  - Generate iid sample from a Pareto model
  - Check sequential standard deviation plot
  - Check normal probability plot

- Result: transformed data is not Gaussian

Recap: Internal Model Consistency

- Relies on traditional modeling approach
- Simple way to exploit high-volume data sets
- Can be used to check underlying assumptions
  - Independent observations
  - Stationarity
- Applicable beyond distributions
  - Stochastic processes
  - Random graph structures
  - Spatio-temporal processes

A Word of Wisdom ...

In my view, even if an accumulation of quick "fixes" were to yield an adequately fitting "patchwork", it would bring no understanding.

– B.B. Mandelbrot, *1997*

The Internet Traffic Story: Part 1

- Early example of measurement-driven Internet research
  - What does real Internet traffic look like?
  - Answer: Go and measure the traffic!
- Traffic data collection
  - Need special-purpose hardware
  - Enormous efforts to check quality of measurements
  - “Measuring the measurer”
- Implications
  - Can get as much high-quality data as one wants
  - Limited by data storage, processing, and analysis capabilities

The Internet Traffic Story: Part 2

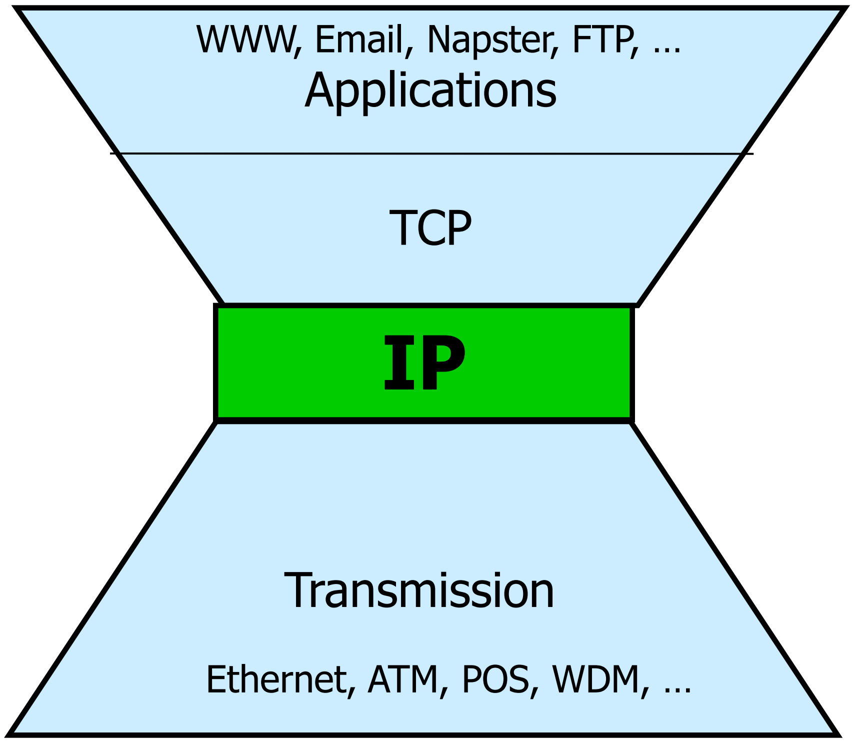
- Modeling
  - So many models, so little insight …
  - How to separate the wheat from the chaff?
- There must be more to modeling than data-fitting
  - Modeling is easy …
  - Insist on models that account for the key structural features of network traffic (e.g., packets belong to flows, connections, sessions)

- Aim for external model consistency …

External Model Consistency: Traffic

- Cross-layer view of models
  - Model for aggregate link traffic (packet-level)
  - Semantic content in packet trace data allows for identification of higher-layer constituents [IP flow, TCP connections, HTTP requests/responses, etc.]
  - Model for aggregate link traffic (higher-layer constituents)
- External model consistency
  - Models should respect the layered network architecture
  - Models should be consistent across layers
  - Models should explain observed phenomena at different layers

Layered Architecture of the Internet



"The Internet hourglass"

On External Model Consistency

- Increased focus on explanatory modeling (as compared to descriptive modeling)
- Yields concrete examples for what is meant by a consistent, mathematically rigorous, networking-based, explanation with supporting measurements
- Resulting models tend to be non-generic but rely on domain-specific details
- Supporting/complementary measurements may not be easily available/accessible

The Internet Traffic Story: Beyond Modeling

- Modeling validation
  - "Good" models point to new types of measurements that can be collected.
  - Use new measurements to check validity of proposed model ("closing-the-loop" argument)
- Now that we have externally consistent traffic models ...
  - Features like **self-similarity** become a non-issue because we understand their root cause, i.e, **heavy-tailed** flows/connections/sessions.
  - Next obvious question: Why "heavy-tailed"?
  - One answer: Optimal web layout results in heavy-tailed HTTP data

On “Closing the Loop”

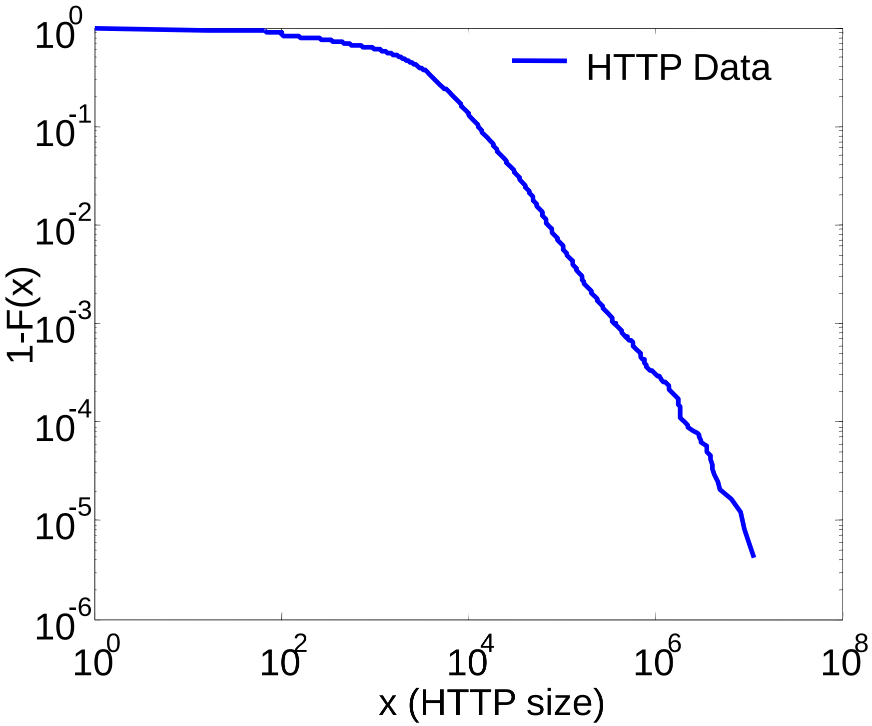
1. Discovery (data-driven)
2. Modeling, subject to internal and external consistency
3. Proposed explanation in terms of elementary concepts or mechanisms (mathematics)
4. Step 3 suggests first-of-its-kind measurements or revisiting existing measurements related to checking the elementary concepts or mechanisms
5. Empirical validation of elementary concepts or mechanisms using the data collected in Step 4

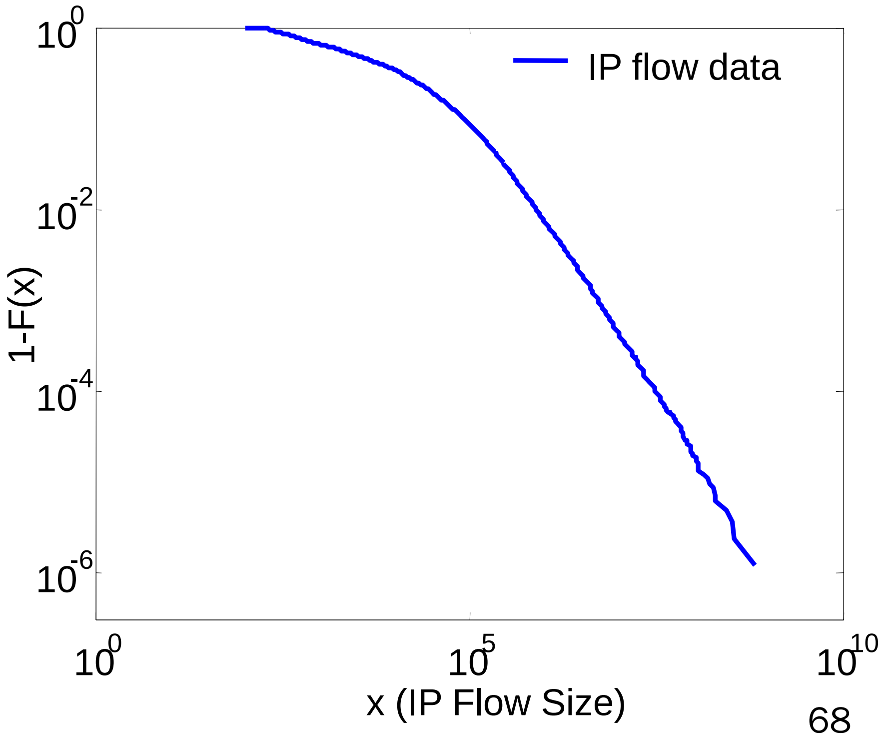
Why "Closing the Loop" is Progress

- Departure from classical "data-fitting"
- Validation is moved to a more elementary or fundamental level
- Fully exploits the context in which measurements are made ("start with data, end with data")
- If successful, provides actual explanation of "emergent" phenomena (new insight)
- Shows inherent limitations and weaknesses of proposed model, suggests further improvements

Recap: Internet Traffic Story

- More than “curve fitting” ...
- More than “is self-similar”...
- Fully consistent with theory and empirical evidence
- Validated by “closing the loop”





Recap: Internet Traffic Story

- By and large, this Internet traffic story has been unsuccessful in turning Internet modeling from an exercise in data fitting into an exercise in reverse-engineering.

- Much of the past work on Internet topology modeling has followed the traditional modeling approach ….

Analysis of Internet Data: Know your Statistics!
Analyzing Low-Quality Internet Data

February 23, 2010

Topics Covered

- Mathematics of heavy-tailed/power-law/scaling distributions
- De-mystifying power-laws
- The virtues of power-laws for low-quality but high-variability data
- A word of caution

A Working Definition

- A distribution function F(x) or random variable X is called heavy-tailed if for some $\alpha > 0$

$$P[X > x] = 1 - F(x) \approx cx^{-\alpha}, x \rightarrow \infty$$

where c>0 and finite

- F is also called a power law or scaling distribution
- The parameter α is called the tail index
- $1 < \alpha < 2$, F has infinite variance, but finite mean
- $0 < \alpha < 1$, the variance and mean of F are infinite
- "Mild" vs "wild" (Mandelbrot): $\alpha \geq 2$ vs $\alpha < 2$

Power laws are ubiquitous

- High variability phenomena abound in natural and man made systems
- Tremendous attention has been directed at whether or not such phenomena are evidence of universal properties underlying all complex systems
- Recently, discovering and explaining power law relationships has been a minor industry within the complex systems literature
- We will use the Internet as a case study to examine what power laws do or don't have to say about its behavior and structure.
- Power laws: Full of sound and fury, signifying nothing? (Strogatz)

First, we review some basic properties about scaling distributions

Power Laws

- Scaling distributions are also called power law distributions.
- We will use notions of power laws, scaling distributions, and heavy tails interchangeably, requiring only that

$$P[X > x] \approx cx^{-\alpha}, \text{ as } x \to \infty \qquad (1)$$

Note that (1) implies

$$\log(P[X > x]) \approx \log(c) - \alpha \log(x)$$

In other words, the CCDF when plotted on log-log scale follows an approximate straight line with slope -α.

Some Simple Constructions

- For U uniform in [0,1], set X=1/U
  - X is heavy-tailed with α=1

- For E (standard) exponential, set X=exp(E)
  - X is heavy-tailed with α=1

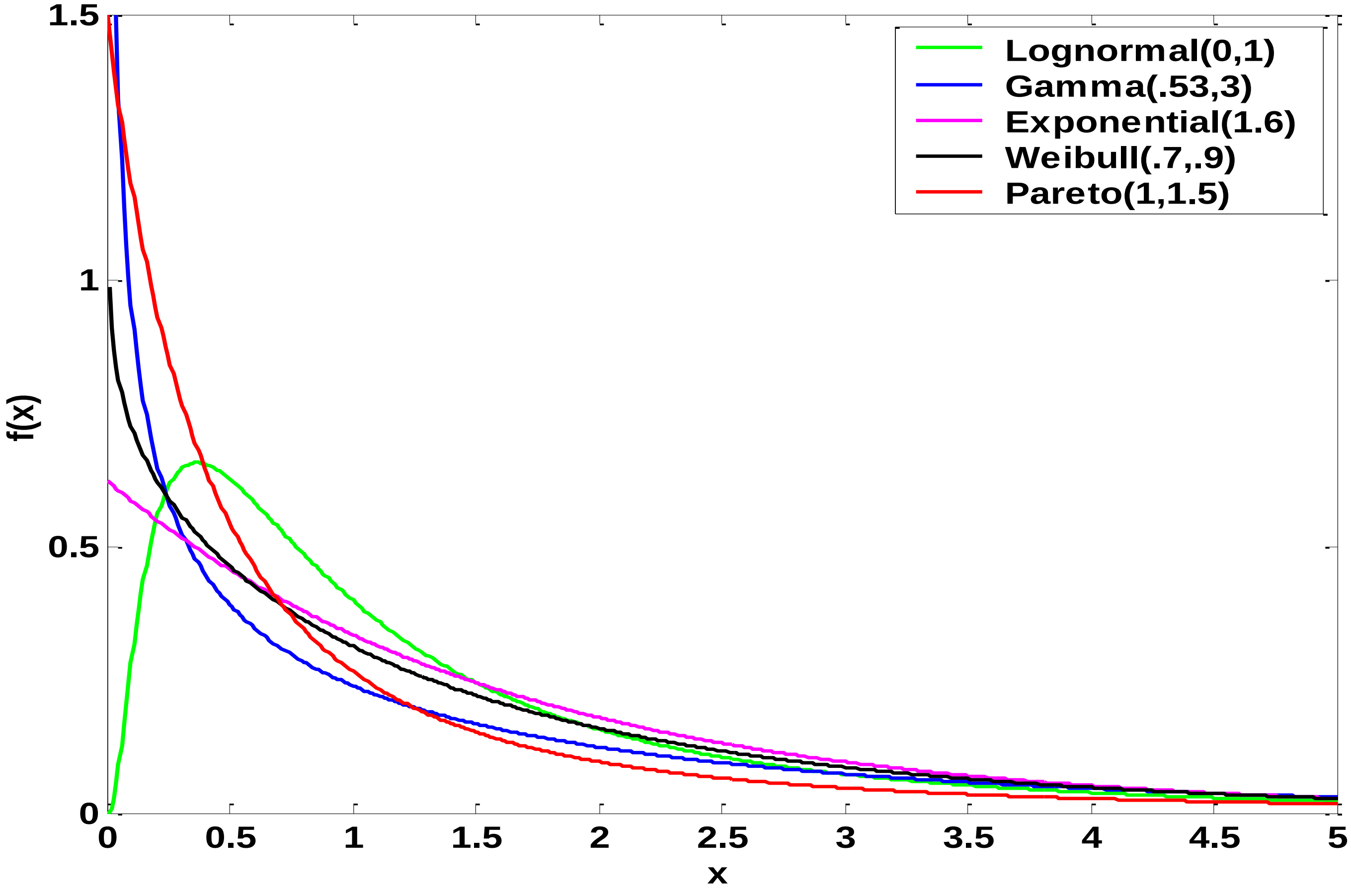
- The mixture of exponential distributions with parameter 1/δ having a (centered) Gamma(a,b) distribution is a Pareto distribution with α=a

- The distribution of the time between consecutive visits of a symmetric random walk to zero is heavy-tailed with α=1/2

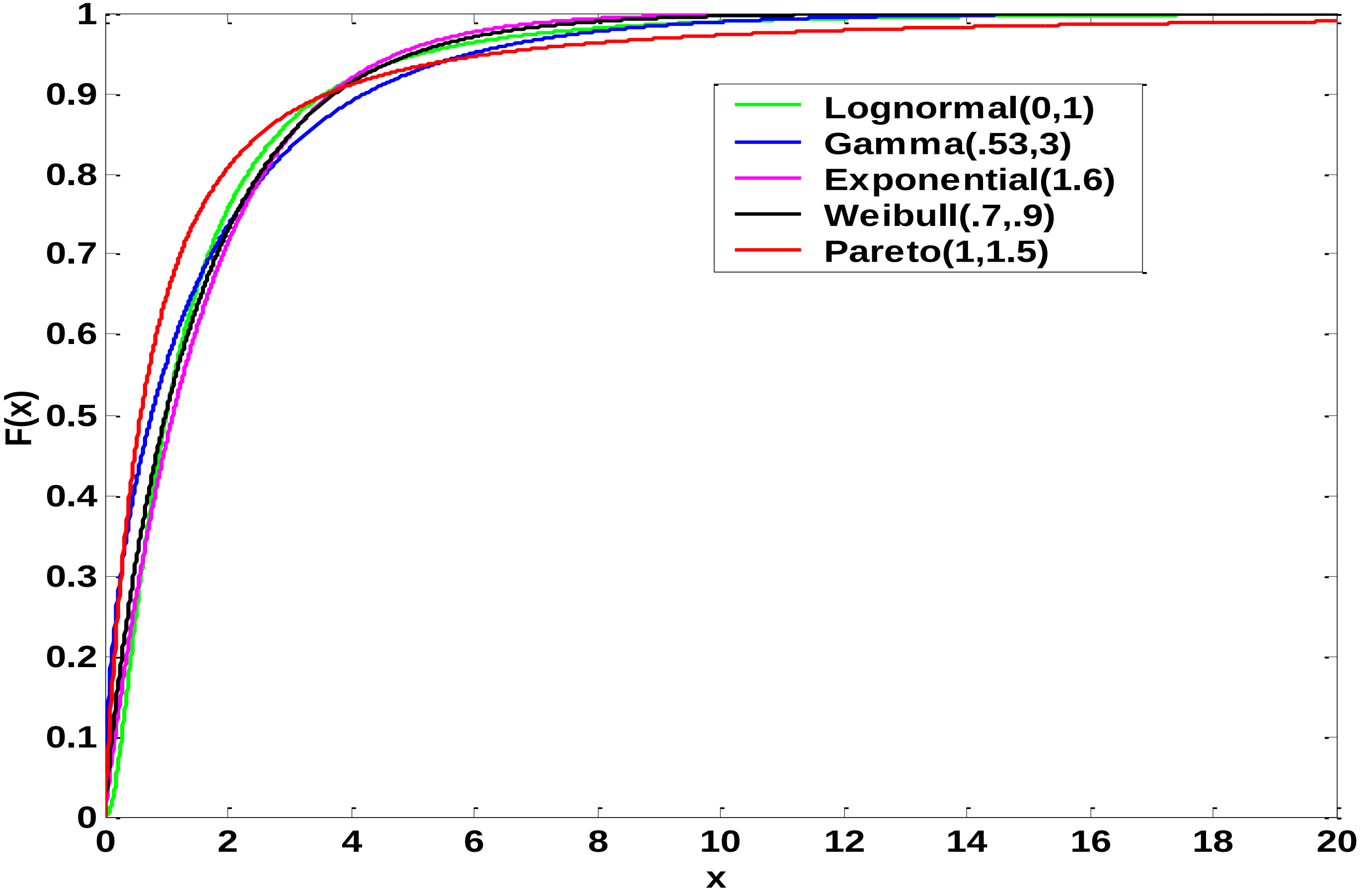
Some Illustrative Examples

- Some commonly-used plotting techniques
  - Probability density functions (pdf)
  - Cumulative distribution functions (CDF)
  - Complementary CDF (CCDF)
- Different plots emphasize different features
  - Main body of the distribution vs. tail
  - Variability vs. concentration
  - Uni- vs. multi-modal

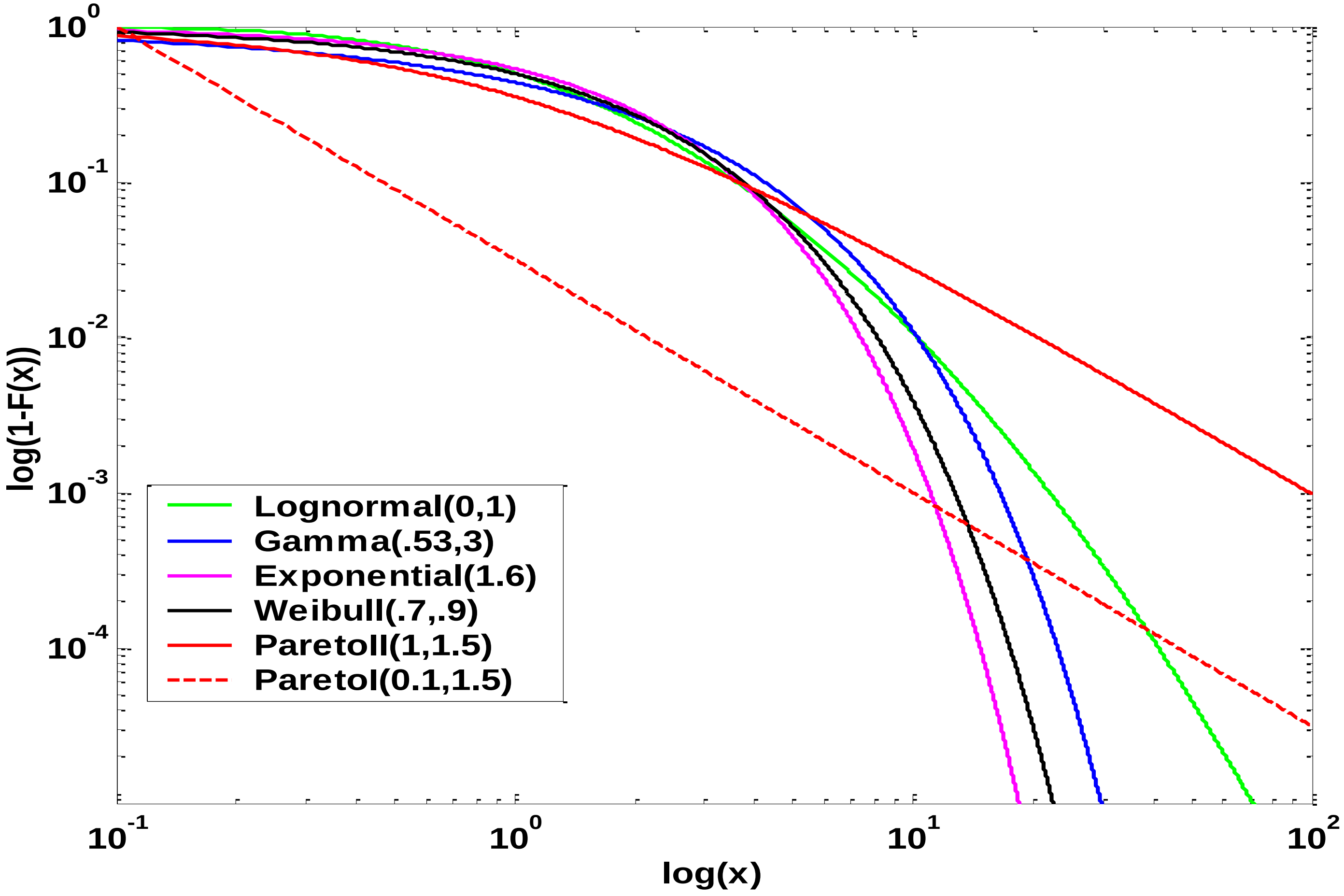
Probability density functions



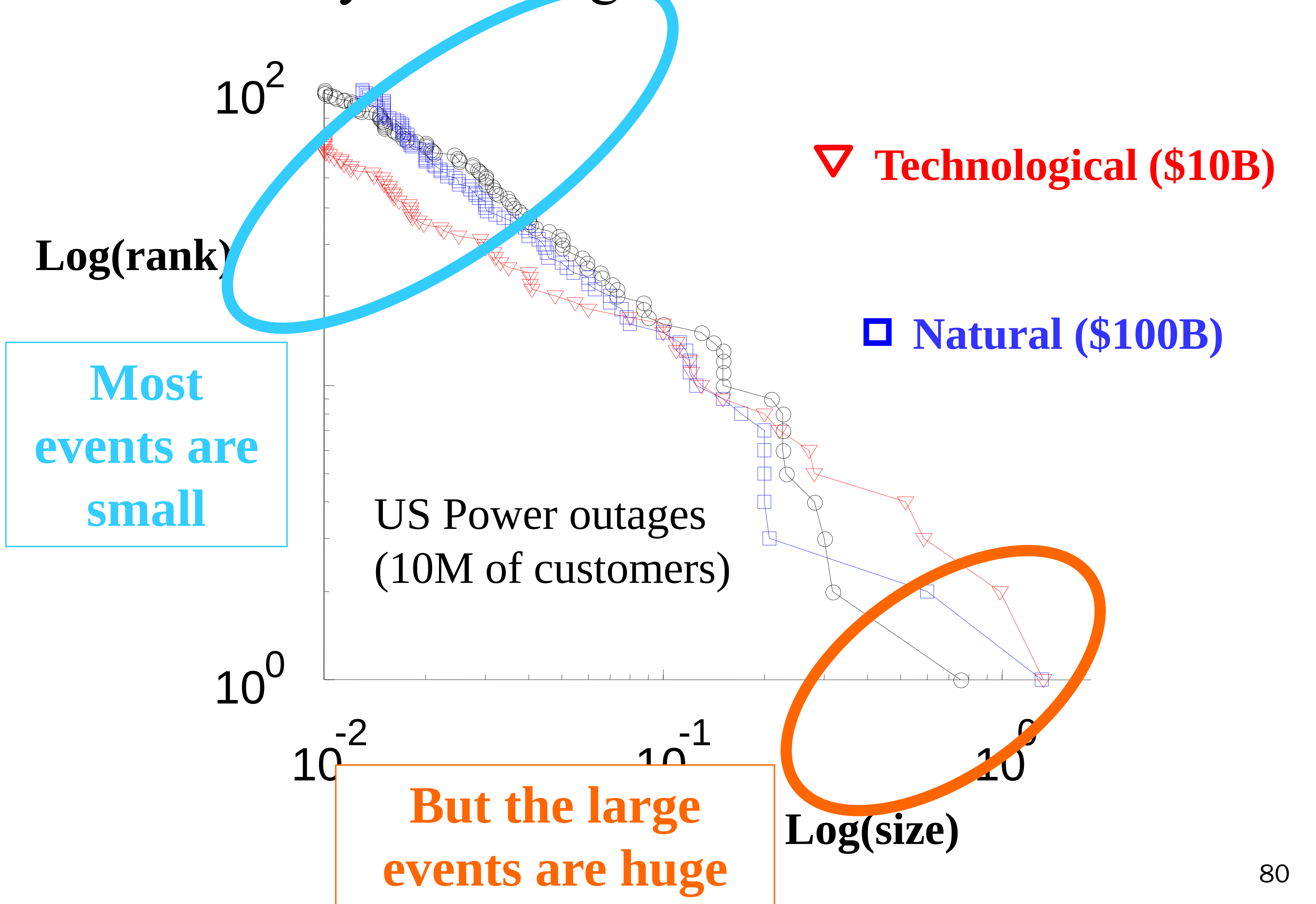
Cumulative Distribution Function

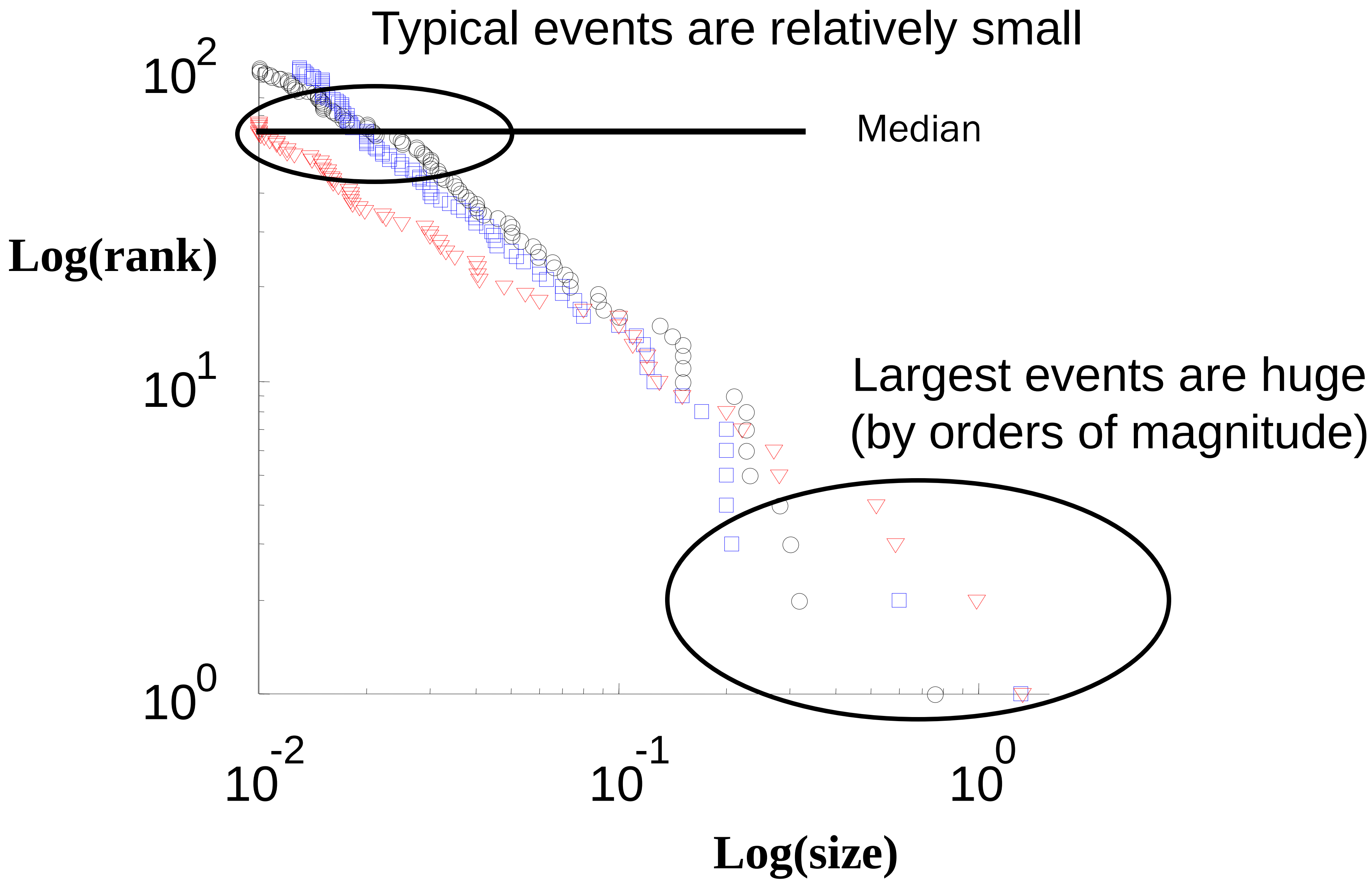


Complementary CDFs



20th Century's 100 largest disasters worldwide





Typical events are relatively small
Median
Largest events are huge
(by orders of magnitude)
Log(rank)
Log(size)
10^2
10^1
10^0
10^{-2}
10^{-1}
10^0

Why "Heavy Tails" Matter ...

- Risk modeling (finance, insurance)
- Load balancing (CPU, network)
- Job scheduling (Web server design)
- Combinatorial search (Restart methods)
- Complex systems studies (SOC, phase transition phenomena)
- Understanding the Internet
  - Behavior (traffic)
  - Structure (connectivity)

Observed/Claimed power law relationships

- Species within plant genera (Yule 1925)
- Mutants in bacterial populations (Luria and Delbrück 1943)
- Economics: income distributions, city populations (Simon 1955)
- Linguistics: word frequencies (Mandelbrot 1997)
- Forest fires (Malamud et al. 1998)
- Earthquakes
- Internet traffic: flow sizes, file sizes, web documents (Crovella and Bestavros 1997)
- Internet topology: node degrees in physical and virtual graphs (Faloutsos et al. 1999)
- Metabolic networks (Barabasi and Oltavi 2004)

Response to Conditioning

- If X is heavy-tailed with index α, then the conditional distribution of X given that $X > w$ satisfies

$$
\begin{aligned}
P[X > x] &= cx^{-\alpha} \\
P[X > x | X > w] &= \frac{P[X > x]}{P[X > w]} \approx c_1 x^{-\alpha}
\end{aligned}
$$

For large values, x is identical to the unconditional distribution $P[X > x]$, except for a change in scale.

- The non-heavy-tailed exponential distribution has conditional distribution of the form

$$
\begin{aligned}
P[X > x] &= e^{-\lambda x} \\
P[X > x | X > w] &= e^{-\lambda (x - w)}
\end{aligned}
$$

The response to conditioning is a change in location, rather than a change in scale.

Mean Residual Lifetime

- An important feature that distinguishes heavy-tailed distributions from non-heavy-tailed counterparts

- For the exponential distribution with parameter λ, mean residual lifetime is constant

$$E(X - x|X > x) = \frac{1}{\lambda},$$

- For a scaling distribution with parameter α, mean residual lifetime is increasing

$$E(X - x|X > x) \approx cx.$$

Key Mathematical Properties of Scaling Distributions

- Response to conditioning - change in scale
- Mean residual lifetime - linearly increasing

Invariance Properties

- Invariant under aggregation
  - Non-classical CLT and stable laws
- (Essentially) invariant under maximization
  - Domain of attraction of Frechet distribution
- (Essentially) invariant under mixture
  - Example: The largest disasters worldwide
- Invariant under marginalization

Linear Aggregation: Classical Central Limit Theorem

- A well-known result
  - $X(1), X(2), \ldots$ independent and identically distributed random variables with distribution function F

    (mean $\mu < \infty$ and variance 1)
  - $S(n) = X(1) + X(2) + \ldots + X(n)$ n-th partial sum

$$\frac{S(n) - n\mu}{n^{1/2}} \rightarrow N(0,1) \text{ as } n \rightarrow \infty$$

- More general formulations are possible
- Often-used argument for the ubiquity of the normal distribution

Linear Aggregation: Non-classical Central Limit Theorem

- A less well-known result
  - $X(1), X(2), \ldots$ independent and identically distributed with common distribution function F that is heavy-tailed with $1 < \alpha < 2$
  - $S(n) = X(1)+X(2)+\ldots+X(n)$ n-th partial sum

$$\frac{S(n) - n\mu}{n^{1/\alpha}} \rightarrow \text{Stable Law, as } n \rightarrow \infty$$

- The limit distribution is heavy-tailed with index α
- More general formulations are possible
- Gaussian distribution is special case when $\alpha = 2$
- Rarely taught in most Stats/Probability courses

Maximization: Maximum Domain of Attraction

- A not so well-known result (extreme-value theory)
  - $X(1), X(2), \ldots$ independent and identically distributed with common distribution function F that is heavy-tailed with $1 < \alpha < 2$
  - $M(n) = \max(X(1), \ldots, X(n))$, n-th successive maxima

$$\frac{M(n)}{n^{1/\alpha}} \rightarrow G, \text{ as } n \rightarrow \infty$$

- G is the Fréchet distribution $\exp(-x^{-\alpha})$
- G is heavy-tailed with index α

Characterizing “Mild” vs. “Wild”

- Aggregation and maximization
  - X(1), X(2), ... independent and identically distributed with common distr. function F
  - Aggregation: $S(n)=X(1)+X(2)+\ldots+X(n)$
  - Maximization: $M(n)=\max\{X(1),X(2),\ldots,X(n)\}$
- Classical case (F is a ”mild” distribution)
  - $M(n)/S(n) \to const, \text{ as } n \to \infty$
- Non-classical case (F is a “wild” distribution)
  - $M(n)/S(n) \to \text{limit law}, \text{as } n \to \infty$

Intuition for "Mild" vs. "Wild"

- The case of "mild" distributions (e.g., const=0)
  - "Evenness" – large values of S(n) occur as a consequence of many of the X(i)'s being large
  - The contribution of each X(i), even of the largest, is negligible compared to the sum
- The case of "wild" distributions
  - "Concentration" – large values of S(n) or M(n) occur as a consequence of a single large X(i)
  - The largest X(i) is dominant compared to S(n)

Weighted Mixture

- A little known result
  - $X(1), X(2), \ldots$ independent random variables having distribution functions F_i that are heavy-tailed with common index $1 < \alpha < 2$, but possibly different scale coefficients c_i
  - Consider the *weighted mixture* $W(n)$ of $X(i)$'s
  - Let p_i be the probability that $W(n) = X(i)$, with $p_1 + \ldots + p_n = 1$, then one can show

    $$P[W(n) > x] \approx c_W x^{-\alpha}, \text{ for large } x$$

    where $c_W = \Sigma\, p_i\, c_i$ is the weighted average of the separate scale coefficients c_i.
- Thus, the weighted mixture of scaling distributions is also scaling with the same tail index, but a different scale coefficient

Multivariate Case: Marginalization

- For a random vector $X \in \mathrm{R}^d$, if all linear combinations $Y = \Sigma_k b_k X_k$ are stable with $\alpha \geq 1$, then X is a stable vector in R^d with index α.
- Conversely, if X is an α-stable random vector in R^d then any linear combination $Y = \Sigma_k b_k X_k$ is an α-stable random variable.

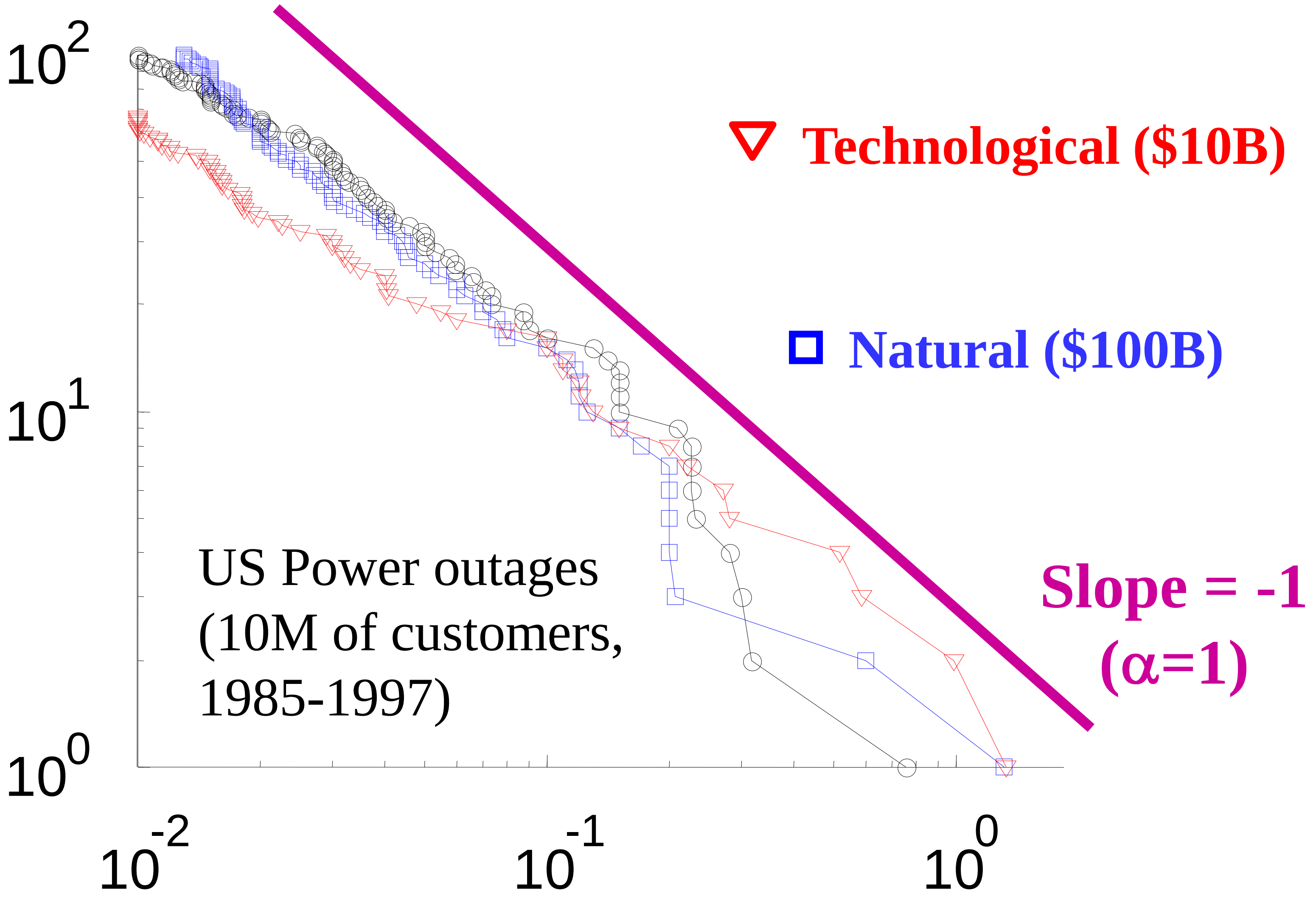
- Marginalization
  - The marginal distribution of a multivariate heavy-tailed random variable is also heavy tailed
  - Consider convex combination denoted by multipliers $b = (0, \ldots, 0, 1, 0, \ldots, 0)$ that projects X onto the k^{th} axis
  - All stable laws (including the Gaussian) are invariant under this type of transformation

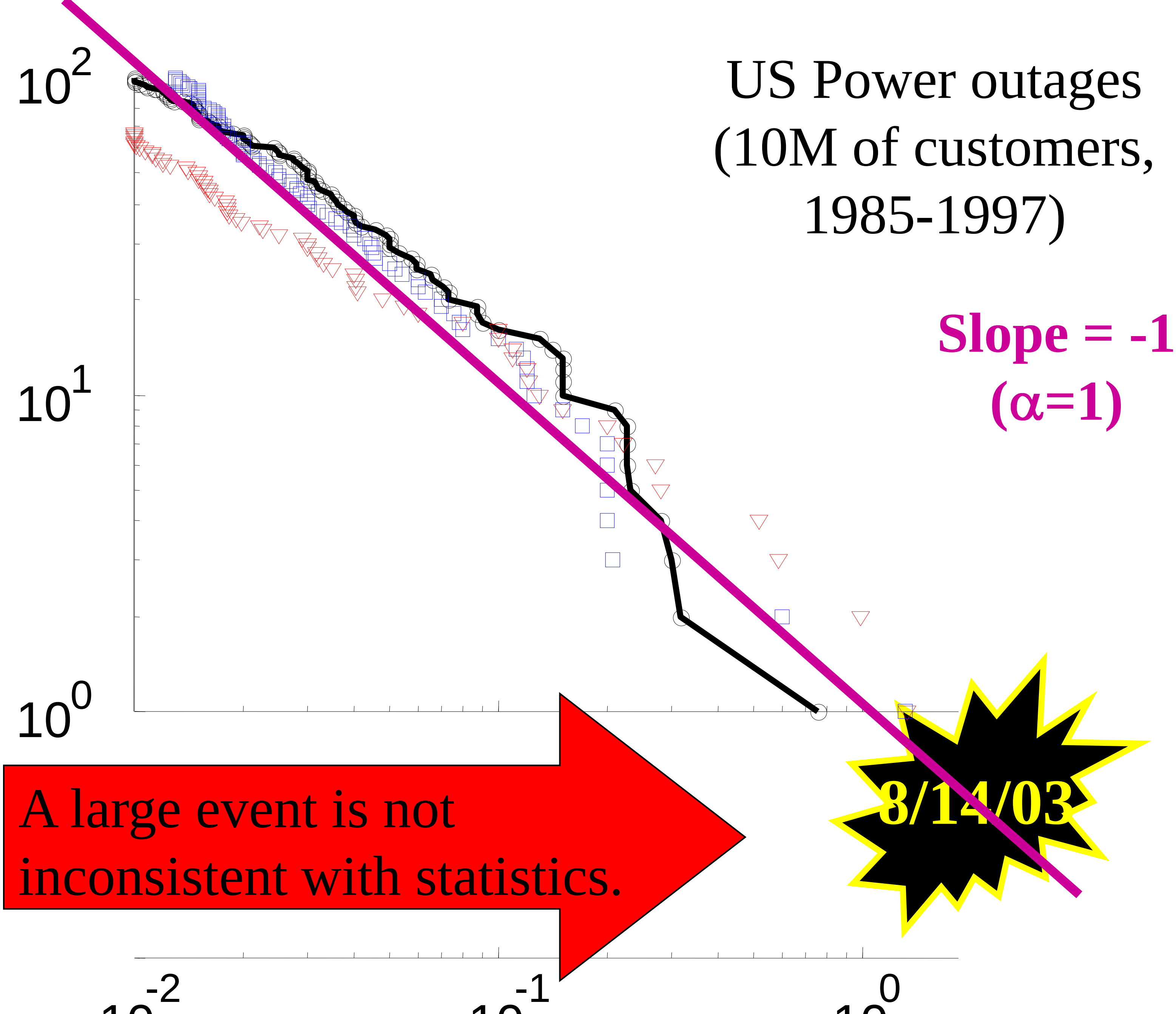
Invariance Properties

	Gaussian Distributions	Scaling Distributions
Aggregation	Yes	Yes
Maximization	No	Yes
Mixture	No	Yes
Marginalization	Yes	Yes

- For low variability data, minimal conditions on the distribution of individual constituents (i.e. finite variance) yields classical CLT
- For high variability data, more restrictive assumption (i.e. right tail of the distribution of the individual constituents must decay at a certain rate) yields greater invariance

20^{th} Century's 100 largest disasters worldwide





US Power outages
(10M of customers,
1985-1997)
Slope = -1
(α=1)
A large event is not
inconsistent with statistics.
8/14/03
10^2
10^1
10^0
10^{-2}
10^{-1}
10^0

Scaling: “more normal than Normal”

- Aggregation, mixture, maximization, and marginalization are transformations that occur frequently in natural and engineered systems and are inherently part of many measured observations that are collected about them.

- Invariance properties suggest that the presence of scaling distributions in data obtained from complex natural or engineered systems should be considered the norm rather than the exception.

- Scaling distributions should not require “special” explanations.

Our Perspective

- Gaussian distributions as the natural null hypothesis for low variability data
  - i.e. when variance estimates exist, are finite, and converge robustly to their theoretical value as the number of observations increases

- Scaling distributions as natural and parsimonious null hypothesis for high variability data
  - i.e. when variance estimates tend to be ill-behaved and converge either very slowly or fail to converge all together as the size of the data set increases

Resilience to Ambiguity

- Scaling distributions are robust under
  - … aggregation, maximization, and mixture
  - … differences in observing/reporting/accounting
  - … varying environments, time periods
- The "value" of robustness
  - Discoveries are easier/faster
  - Properties can be established more accurately
  - Findings are not sensitive to the details of the data gathering process

On the Ubiquity of Heavy Tails

- Heavy-tailed distributions are attractors for averaging (e.g., non-classical CLT), but are the only distributions that are also (essentially) invariant under maximizing and mixing.
- Gaussians ("normal") distributions are also attractors for averaging (e.g., classical CLT), but are not invariant under maximizing and mixing
- This makes heavy tails more ubiquitous than Gaussians, so no "special" explanations should be required …

Looking ahead ...

- Main objective of current empirical studies

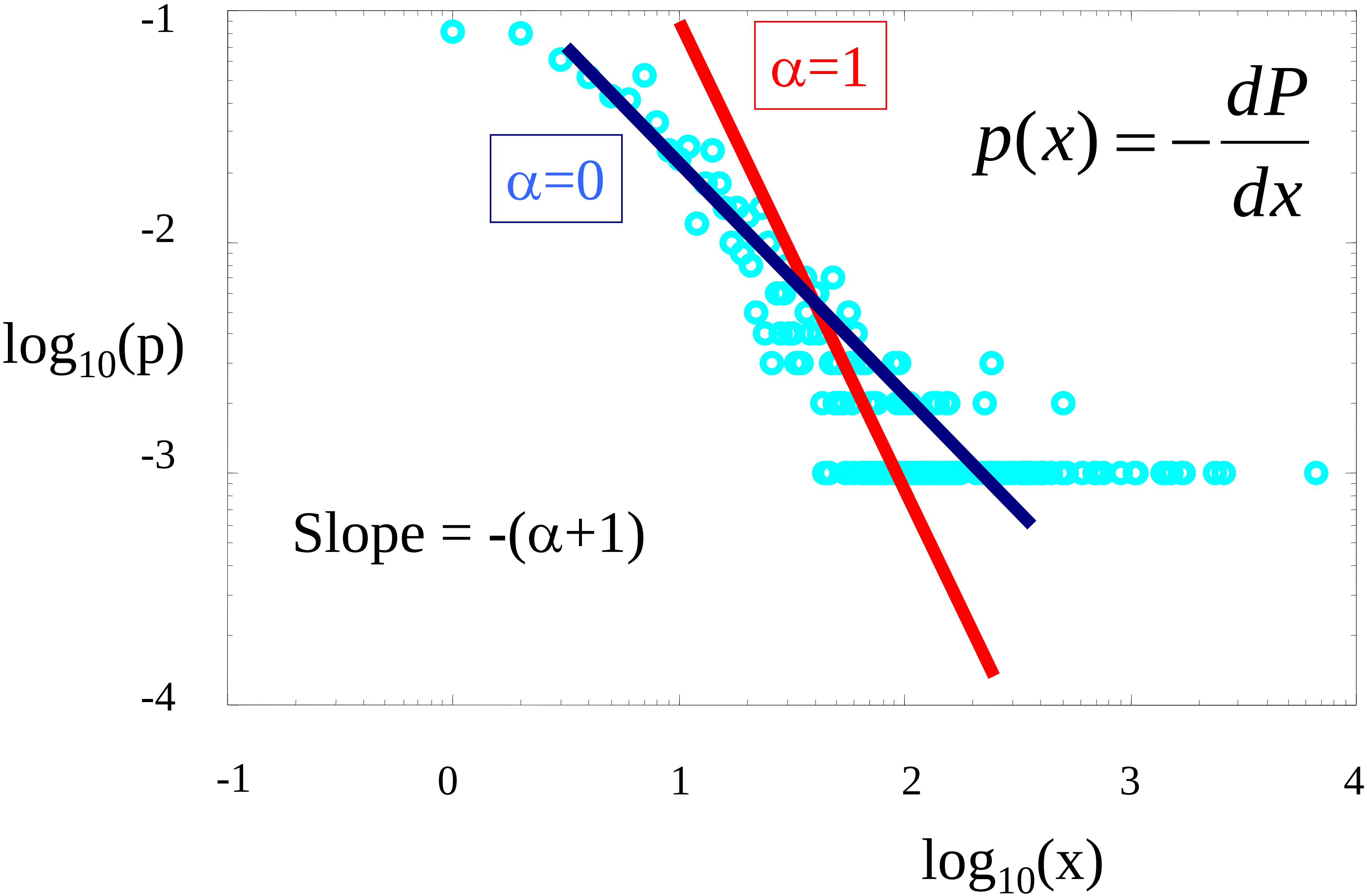
  "The observations are consistent with model/distribution X, but are not consistent with model/distribution Y."

- Requirement for future empirical studies

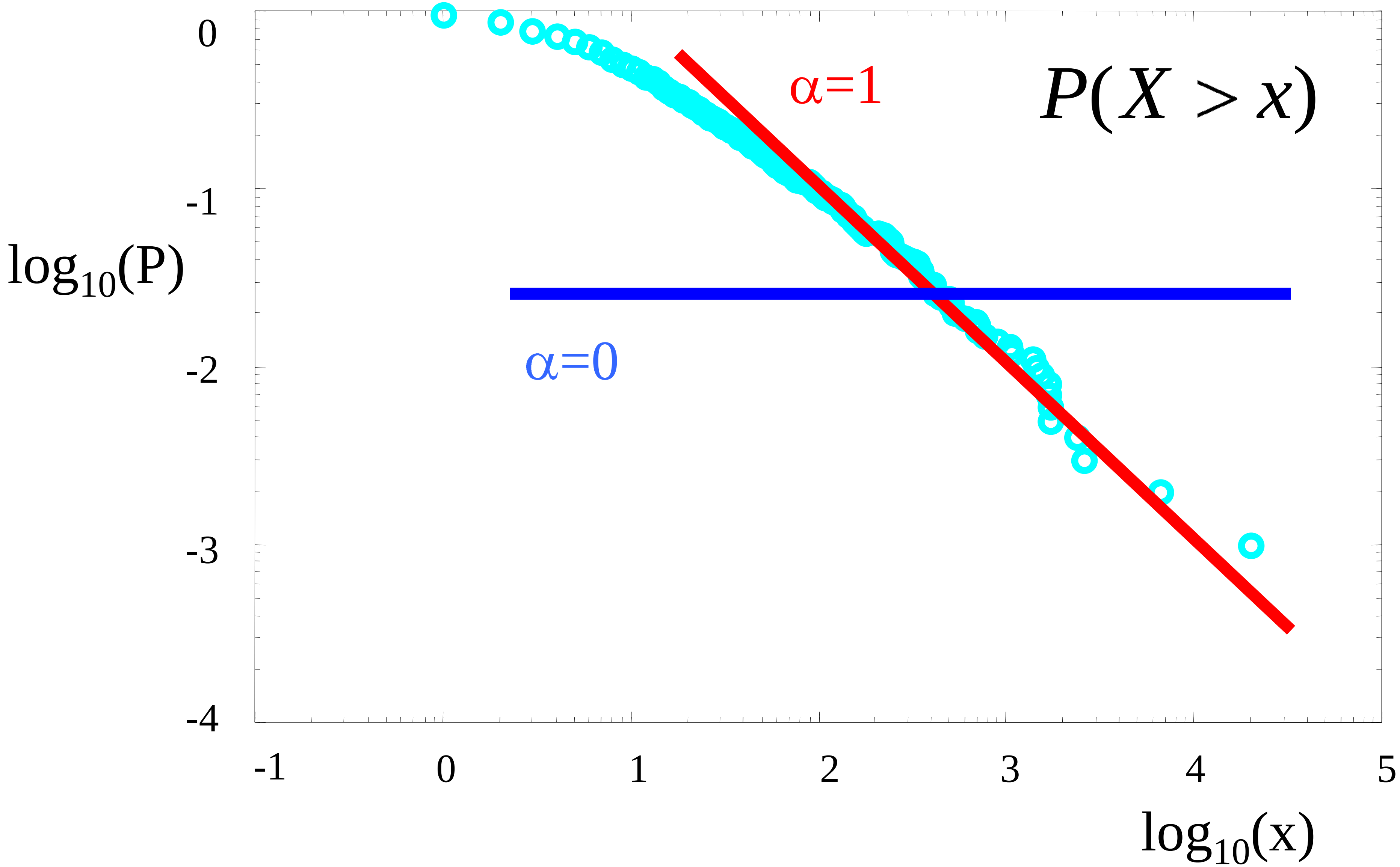
  "The observations are consistent with model/distribution X, and X is not sensitive to the methods of measuring and collecting the observations."

Some Words of Caution ...

- Not every “straight-looking” log-log plot means “heavy tails”!
- Never use frequency plots to infer heavy tails – even though physicists do it all the time!



NEVER use frequency plots on log-log scale!

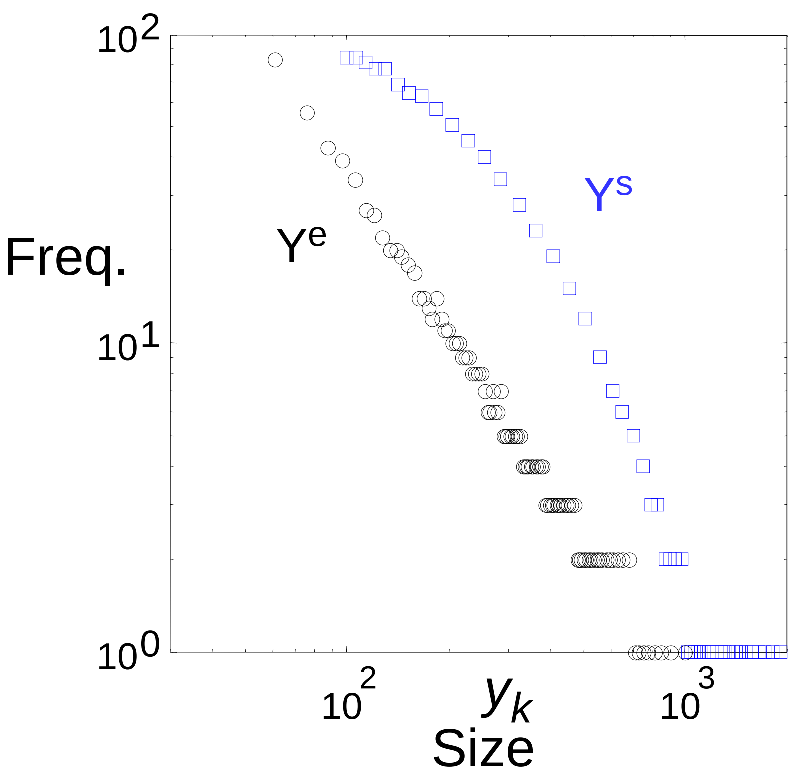


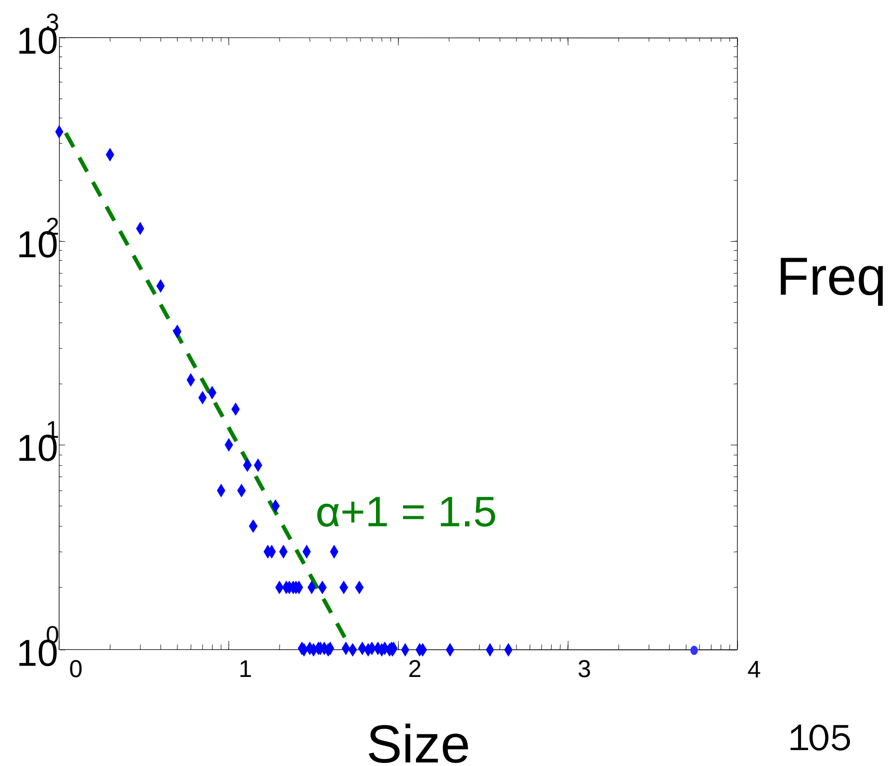
ALWAYS use CCDF plots on log-log scale!

Observation #3: If you want your data to exhibit a power law behavior, use size-frequency plots!

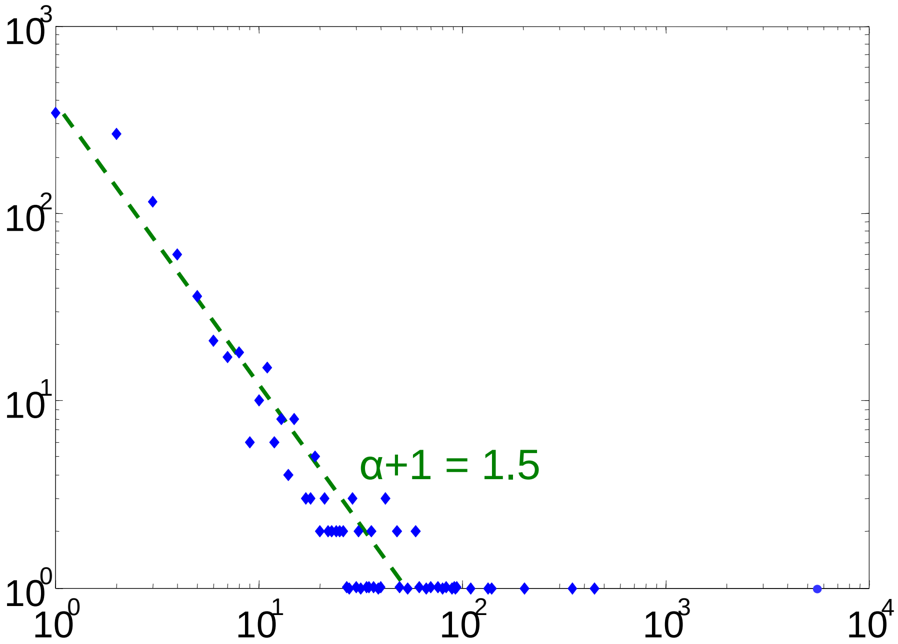
Given: Samples from an exponential distribution
Want: Claim power law behavior
Recipe: Use size-frequency plots!

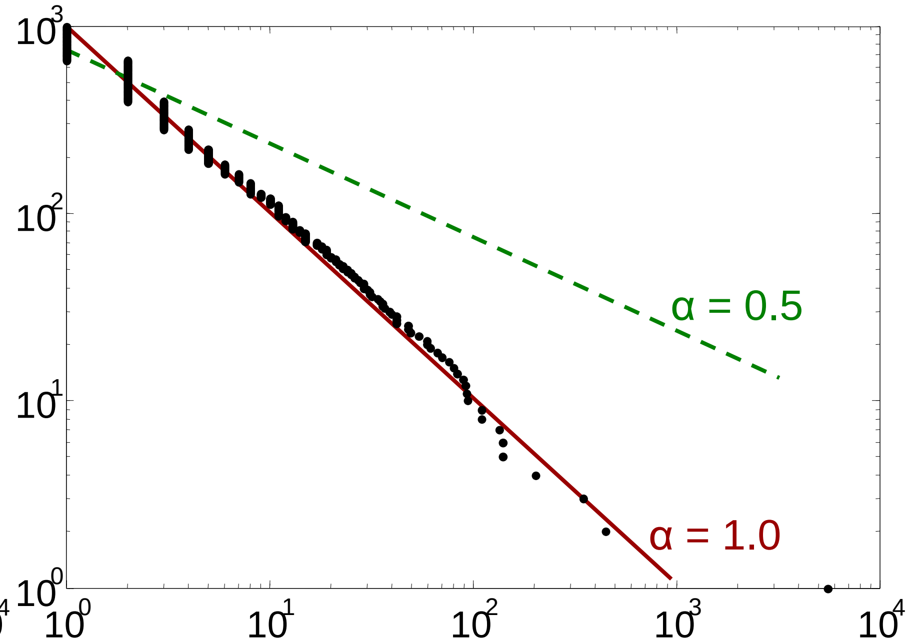
Given: Samples from a Pareto distribution with α=1.0
Want: Claim power law with α=1.5
Recipe: Use size-frequency plots!

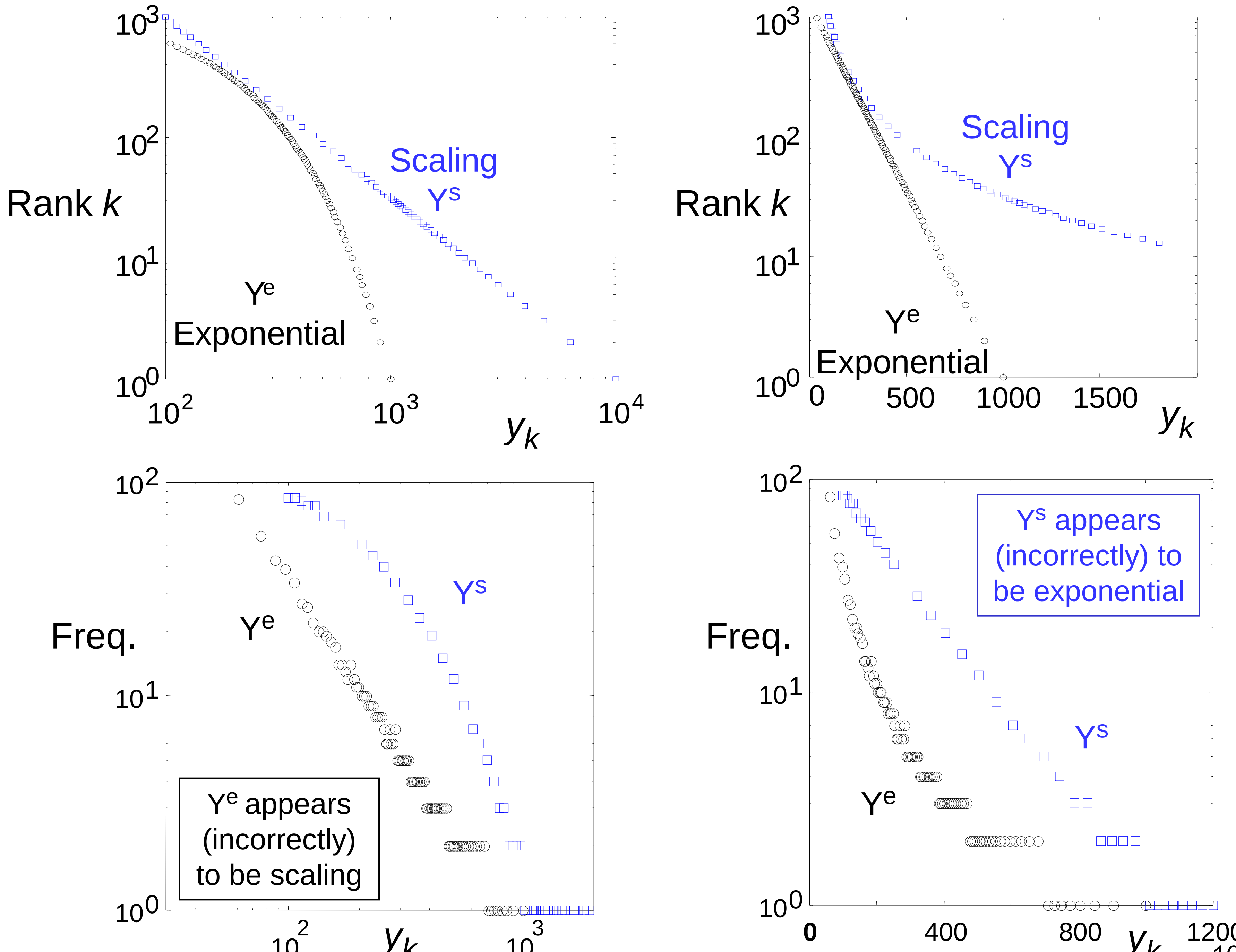




Size-Frequency vs. Size-Rank Plots
or
Non-cumulative vs. Cumulative



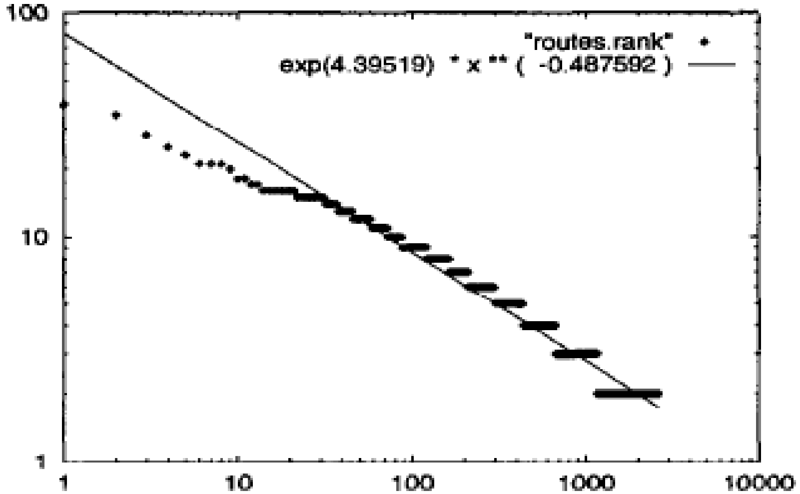




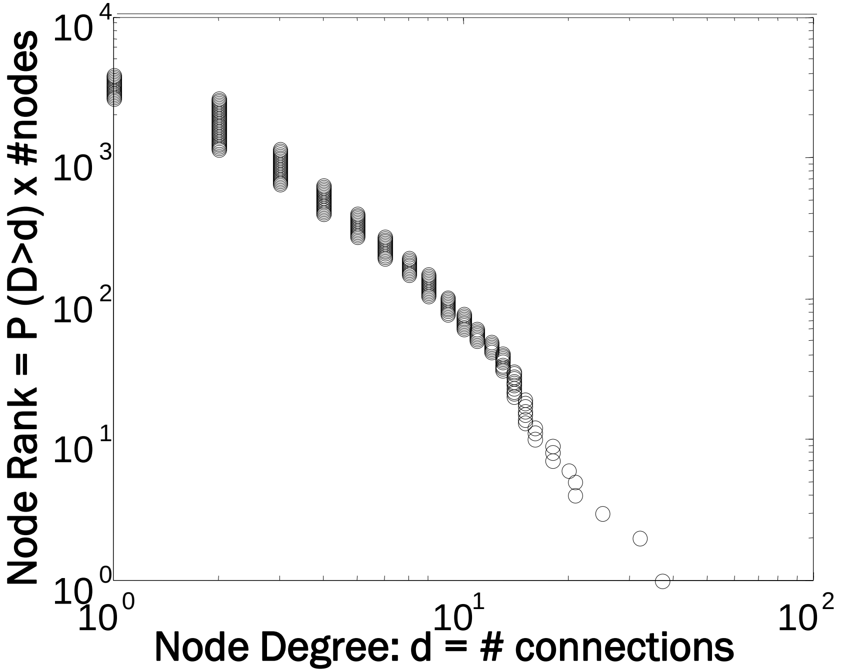
Rank k
Scaling
Y^s
Y^e
Exponential
y_k
Freq.
Y^e appears (incorrectly) to be scaling
Y^s appears (incorrectly) to be exponential

Example 1: Claimed Power Laws in Router-Level Topology

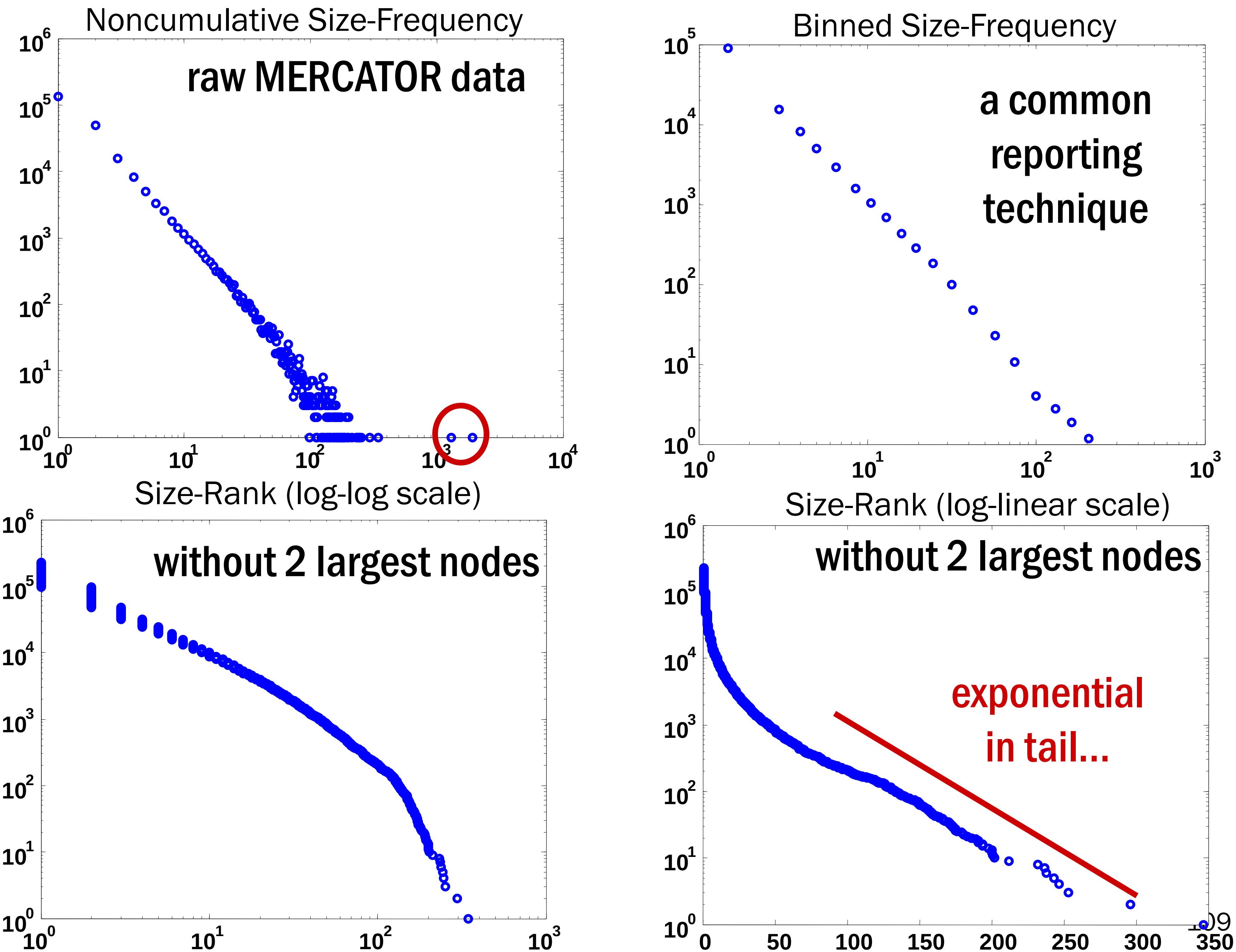
Source: Faloutsos et al. (1999)



(b) Rout-95



Example 2: Claimed power law of Internet connectivity data!



Beyond Traditional Data Analysis

- Requirement 1: Internal Model Consistency
  - Exploit high volume of available data
  - Learn from Mandelbrot and Tukey
  - Example: Understanding HTTP and IP data

- Requirement 2: External Model Consistency
  - Exploit rich semantic of available data
  - Learn more from Mandelbrot and Cox
  - Example: Understanding self-similar Internet traffic

- Requirement 3: Resilience to Ambiguous Data
  - High variability to the rescue
  - Again, look up Mandelbrot
  - Example: Understanding Internet topology data

Internet Modeling:
From Data-Fitting to Reverse-Engineering

February 23, 2010

The Internet as a Case Study

- To the user, it creates the illusion of a simple, robust, homogeneous resource enabling endless varieties and types of technologies, physical infrastructures, virtual networks, and applications (heterogeneous).
- Its complexity is starting to approach that of simple biological systems
- Our understanding of the underlying technology together with the ability to perform detailed measurements means that most conjectures about its large-scale properties can be unambiguously resolved, though often not without substantial effort.

Internet Topology as a Case Study

- How to make complex systems still complex but experimentally accessible?
- Importance/interpretation of high variability in complex systems – de-mystifying power-laws
- Modeling debate: design vs. randomness
- Understanding the “robust, yet fragile” aspects of the Internet
- “Closing the loop” between modeling and analysis

Why Modeling Network Topology?

- Performance evaluation of protocols
- Provisioning
  - Topology constrains the applications and services that run on top of it
- Understanding large-scale properties
  - Reliability and robustness to accidents, failures, and attacks on network components
- Insight into other network systems
  - To the extent that the network model is “universal”
- Getting at the architecture of a system by understanding the various connectivity structures that are exposed

State-of-the-art Topology Modeling

- Direct inspection generally not possible
  - Measurement-driven research activity
- Recent trend
  - Generative models follow empirical measurement studies
- But…
  - So many things to measure
  - Incredible variability in so many aspects
  - How to determine what matters?

- Our main focus is on router-level topology

Recap: What "Network Science" says about the Internet

- Measurements
  - Router-level: large-scale traceroute experiments
  - AS-level: BGP-based, traceroute-based, WHOIS
  - WWW: large-scale web crawling experiments
- Inference
  - (Exclusive) focus on node degree distribution
  - Inferred node degree distributions follow a power law
- Modeling
  - Preferential attachment-type growth model
    - Incremental growth
    - Preferential attachment: $p(k) \approx$ degree of node k
  - There exist many variants of this basic PA model

Recap: What "Network Science" says about the Internet (cont)

- Key features of PA-type models
  - Randomness enters via attachment mechanism
  - Exhibit power law node degree distributions with or without exponential cutoffs
- Model validation
  - The model "fits the data ..."
  - Reproduces observed node degree distribution
- Highly publicized claims about Internet topology
  - High-degree nodes form a hub-like core
  - Fragile/vulnerable to targeted node removal
  - Achilles' heel
  - Zero epidemic threshold

Basic Question

Do the available Internet-related connectivity measurements and their analysis support the sort of claims that can be found in the existing complex networks literature?

Short Answer: No!

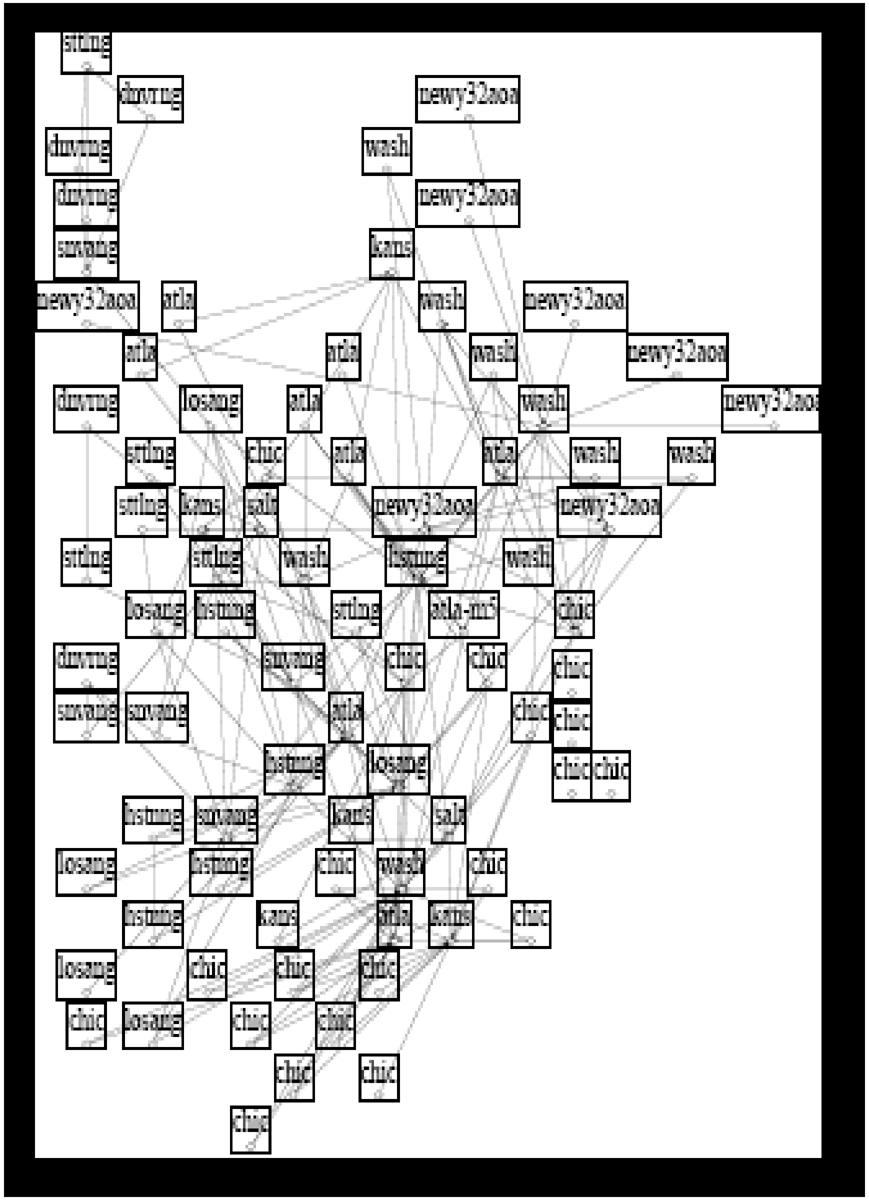
Long Answer: No, because ...

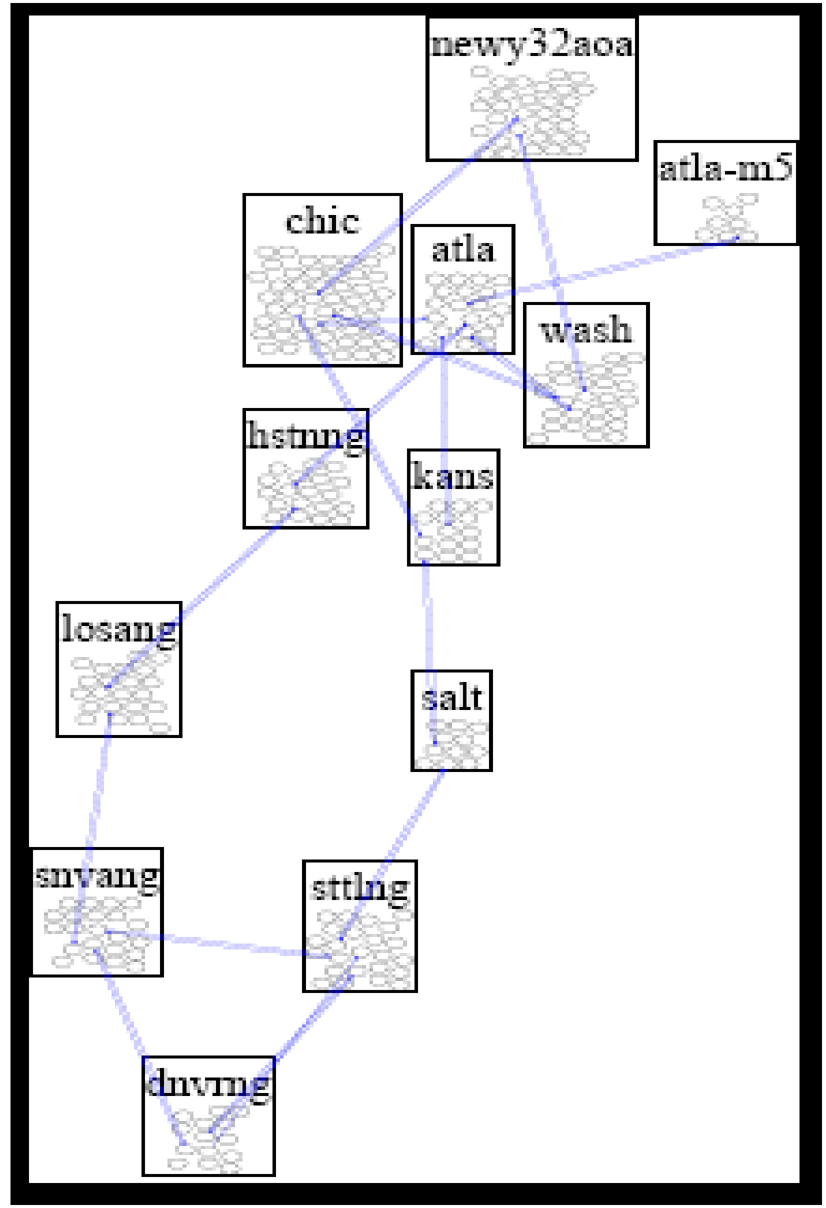
- Lack of data hygiene
- Lack of scientific hygiene (power-laws)
- Lack of critical model validation

Recap: Part I (Know your data!)

- The currently available traceroute measurements are in general of insufficient quality little to make any scientifically sound inferences about the Internet's router-level connectivity
- Main problems
  - IP alias resolution problem (i.e., mapping IP router interfaces to the correct routers)
  - Cannot trace through opaque Layer-2 clouds

IP Alias Resolution Problem for Abilene (thanks to Adam Bender)





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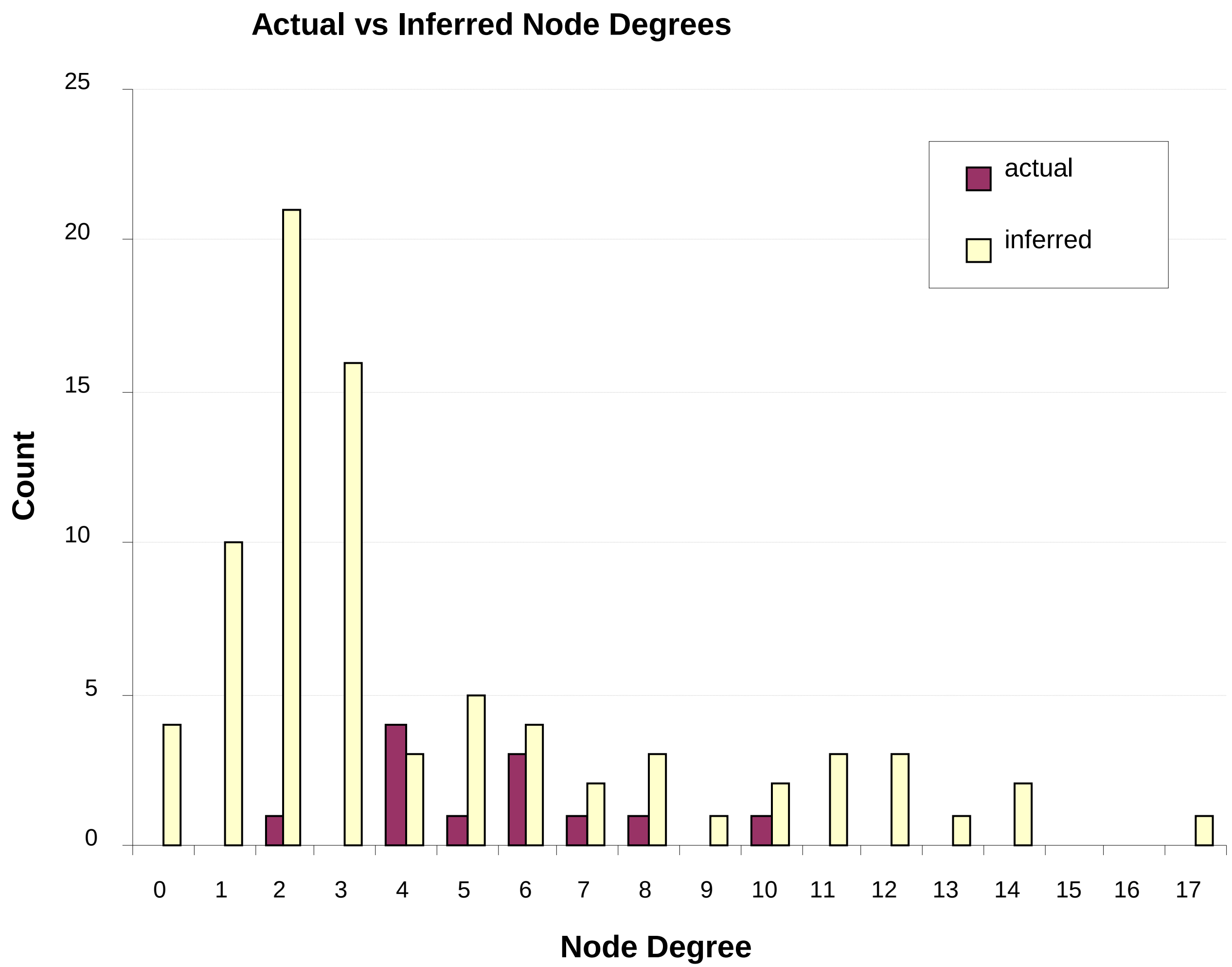
- Main implications
  - The large node degrees are wrong
  - The remaining node degrees are unreliable


Actual vs Inferred Node Degrees
Count
Node Degree
actual
inferred
0
5
10
15
20
25
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

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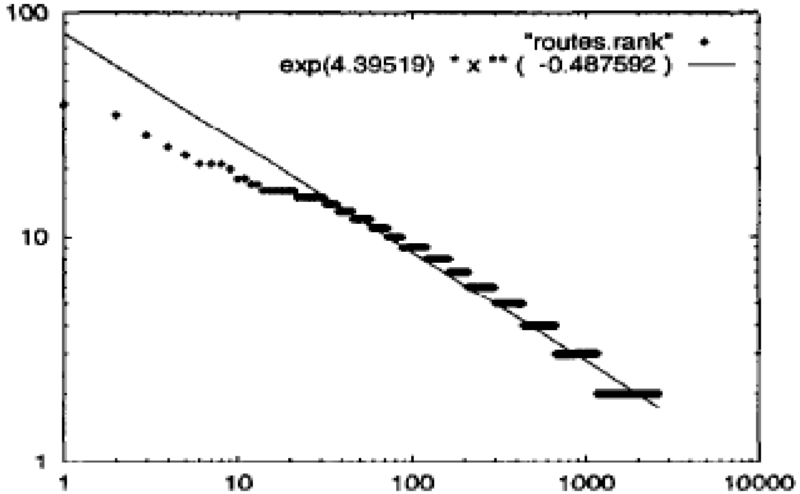
  - Cannot trace through opaque Layer-2 clouds
- Main implications
  - The large node degrees are wrong
  - The remaining node degrees are unreliable
- **Power-law claim for the router-level Internet**
  - **(White) lie, damned lie**

Recap: Part II (Know your Statistics!)

- Even if the currently available traceroute measurements were of sufficient quality to be used for inferring the Internet's router-level connectivity (but recall, this is really stretching things!) …
- Main problems
  - The level of statistical analysis applied to conclude power-law behavior is dismal
  - Dominated by techniques that "guarantee" power-law behavior even if it doesn't exist

Example 1: Claimed Power Laws in Router-Level Topology

Source: Faloutsos et al. (1999)



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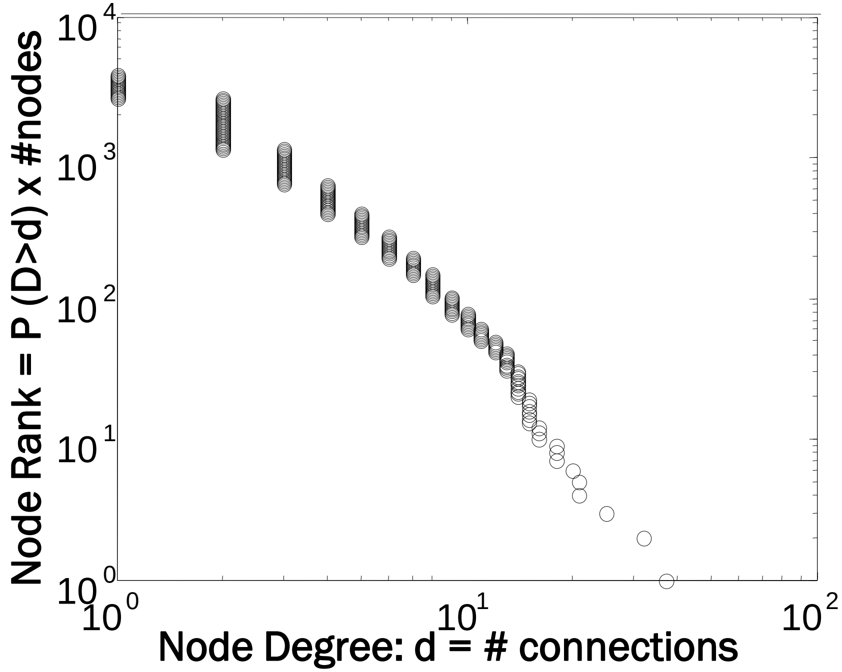

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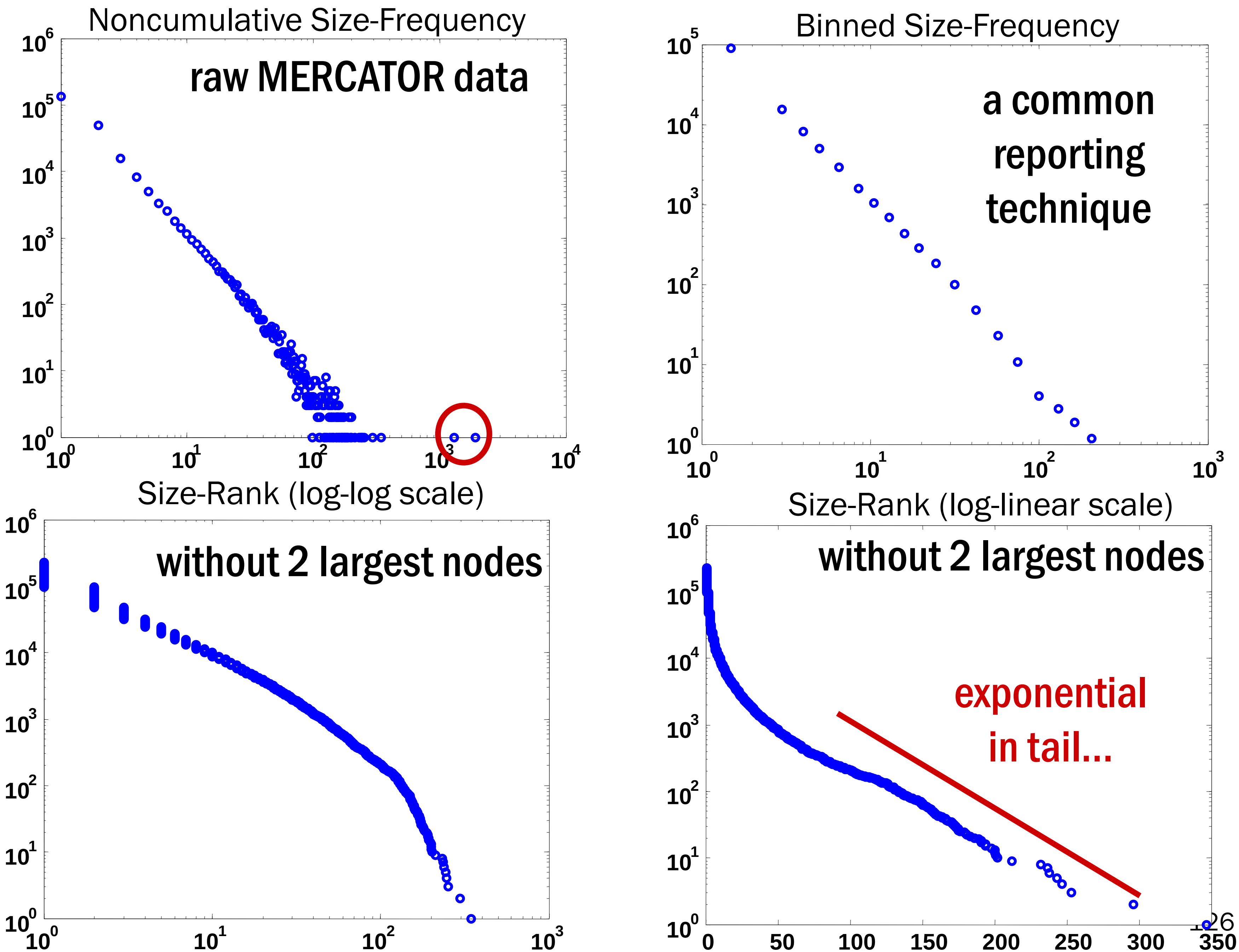

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- Main problems
  - The level of statistical analysis applied to conclude power-law behavior is dismal
  - Dominated by techniques that "guarantee" power-law behavior even if it doesn't exist
- **Power-law claim for the router-level Internet**
  - **Textbook example of "how to lie with statistics …"**

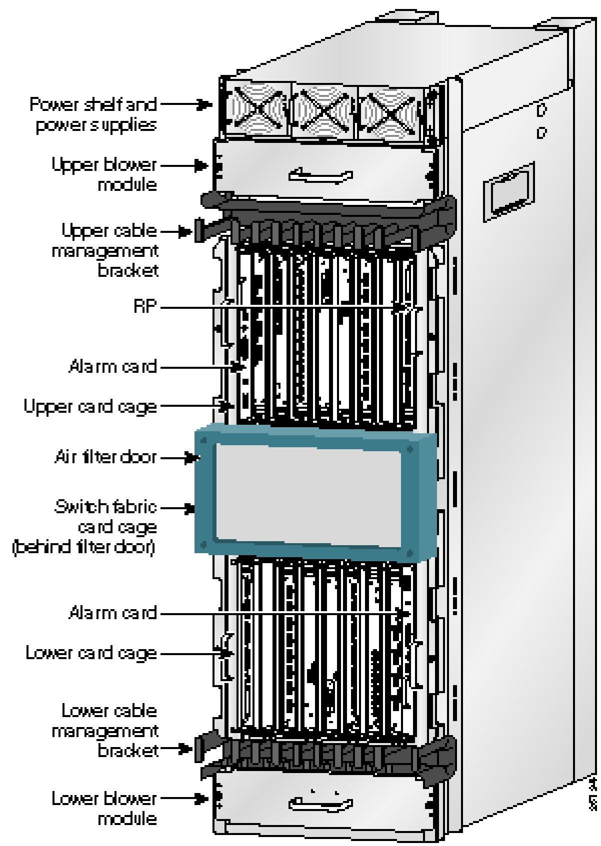
Model Validation: A "hands-on" Approach

- Mathematical Modeling 101
  - For one and the same observed phenomenon, there are usually many different explanations/models
  - All models are wrong, but some are "really" wrong ...
- Model validation ≠ data fitting
  - The ability to reproduce a few graph statistics does not constitute "serious" model validation
  - Which of the observed properties does a proposed model have to satisfy before it is deemed "valid"?
- What constitutes "serious" model validation?
  - There is more to networks than connectivity
  - When "nodes" and "links" have specific meaning ...
  - What do real networks look like?

Cisco 12000 Series Routers

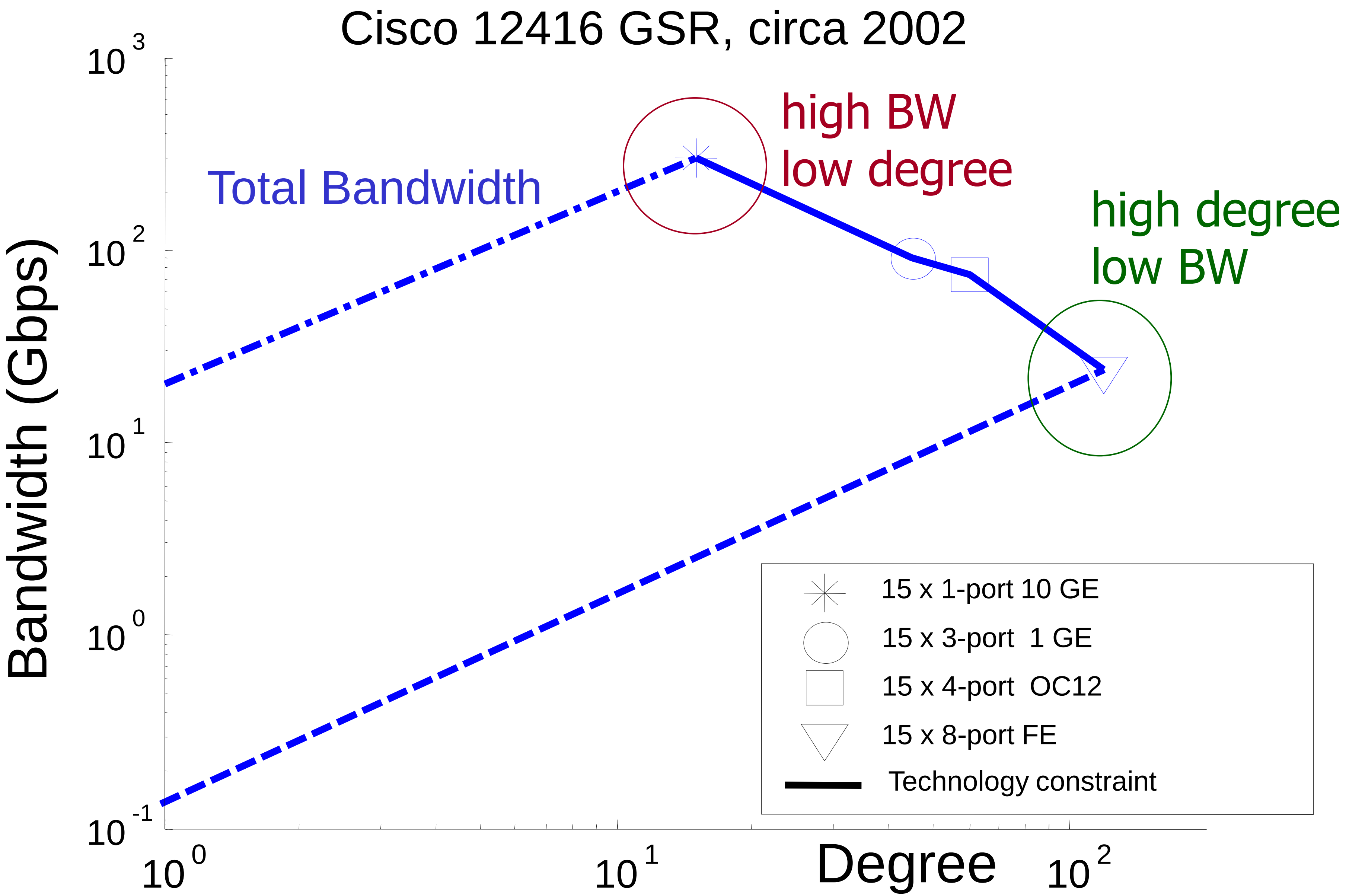
- Modular in design, creating flexibility in configuration.
- Router capacity is constrained by the number and speed of line cards inserted in each slot.

Chassis	Rack size	Slots	Switching Capacity
12416	Full	16	320 Gbps
12410	1/2	10	200 Gbps
12406	1/4	6	120 Gbps
12404	1/8	4	80 Gbps

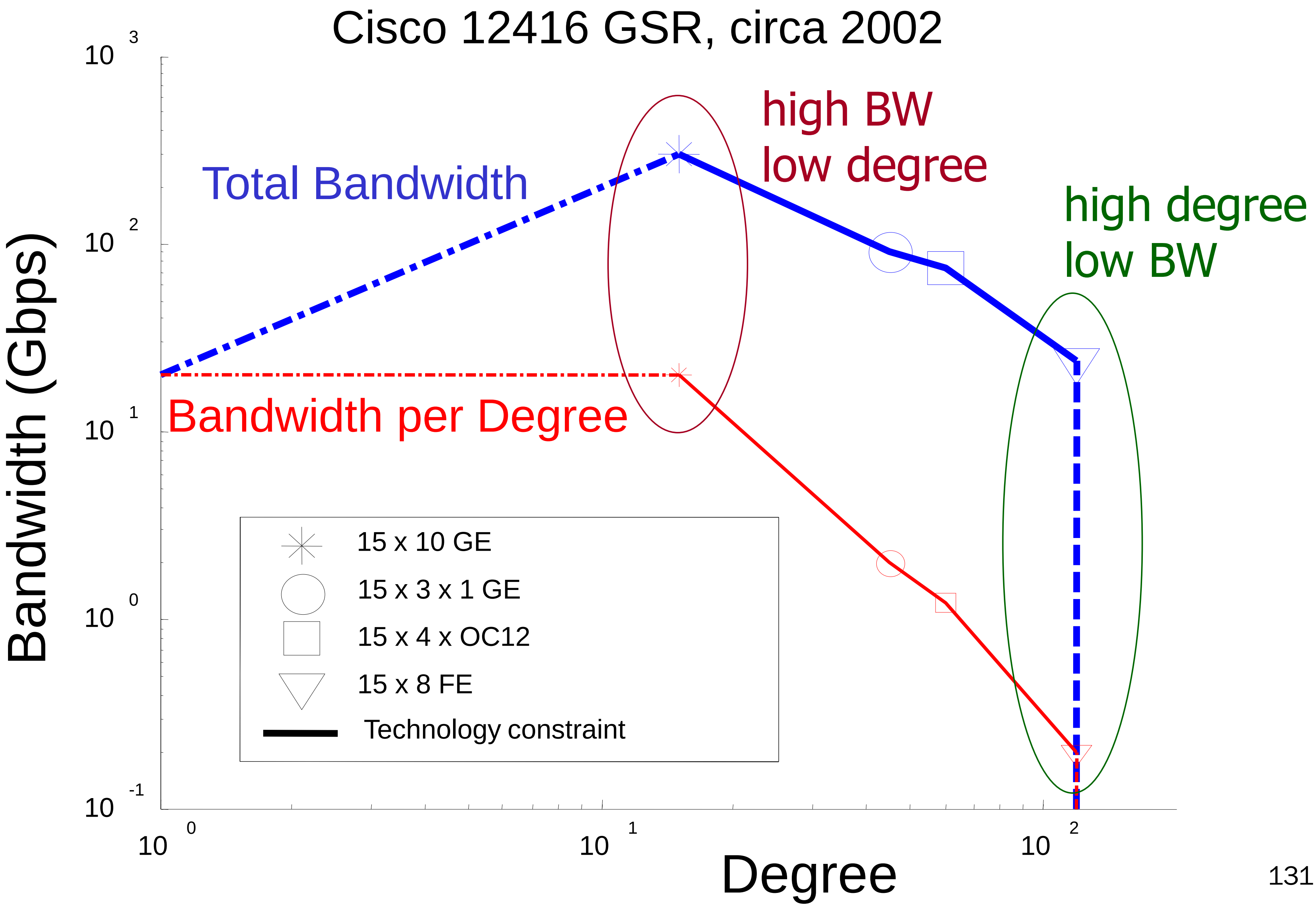


Source: www.cisco.com

Router Technology Constraint

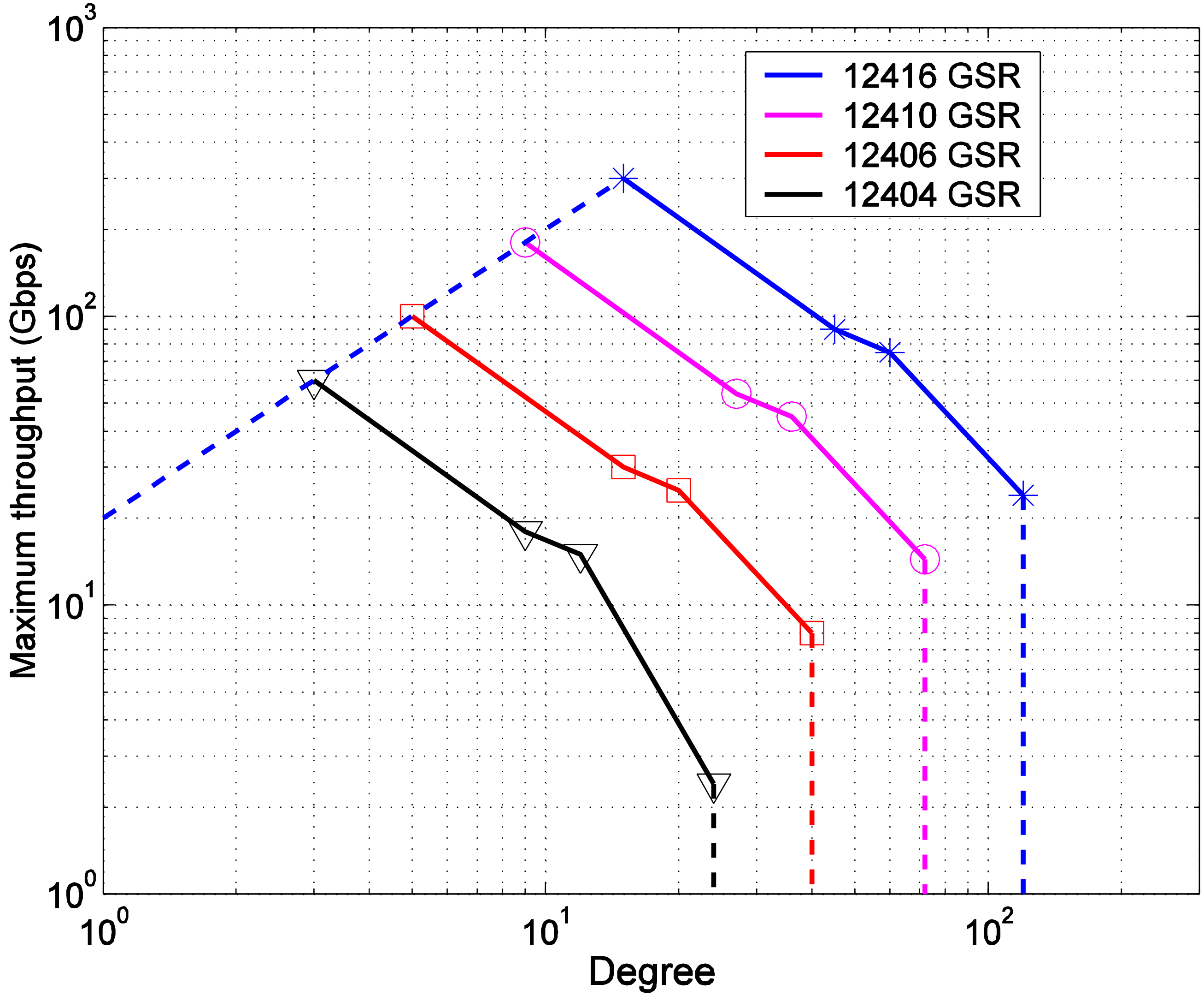


Router Technology Constraint



Cisco 12000 Series Routers

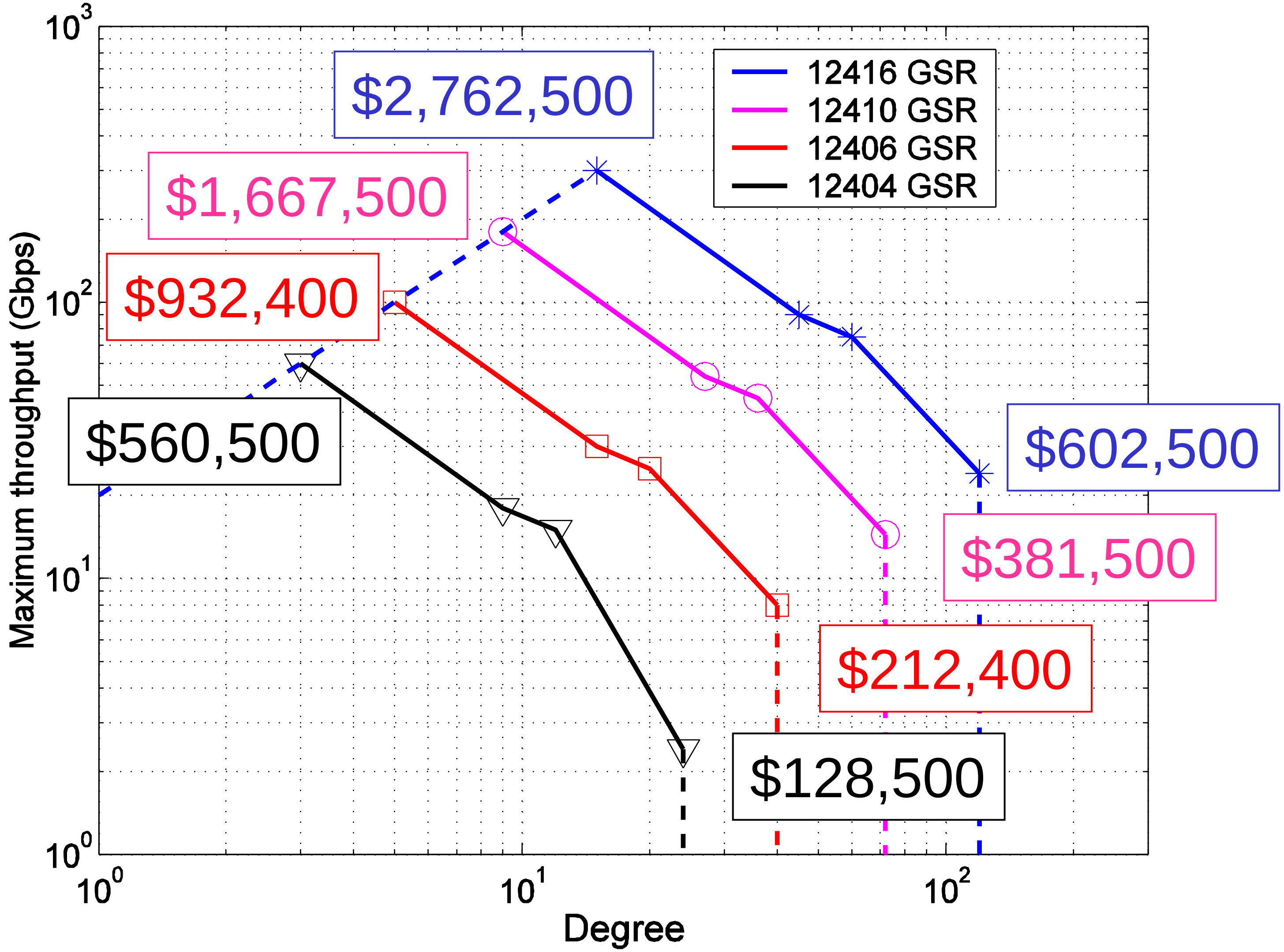
Technology constrains the number and capacity of line cards that can be installed, creating a feasible region.



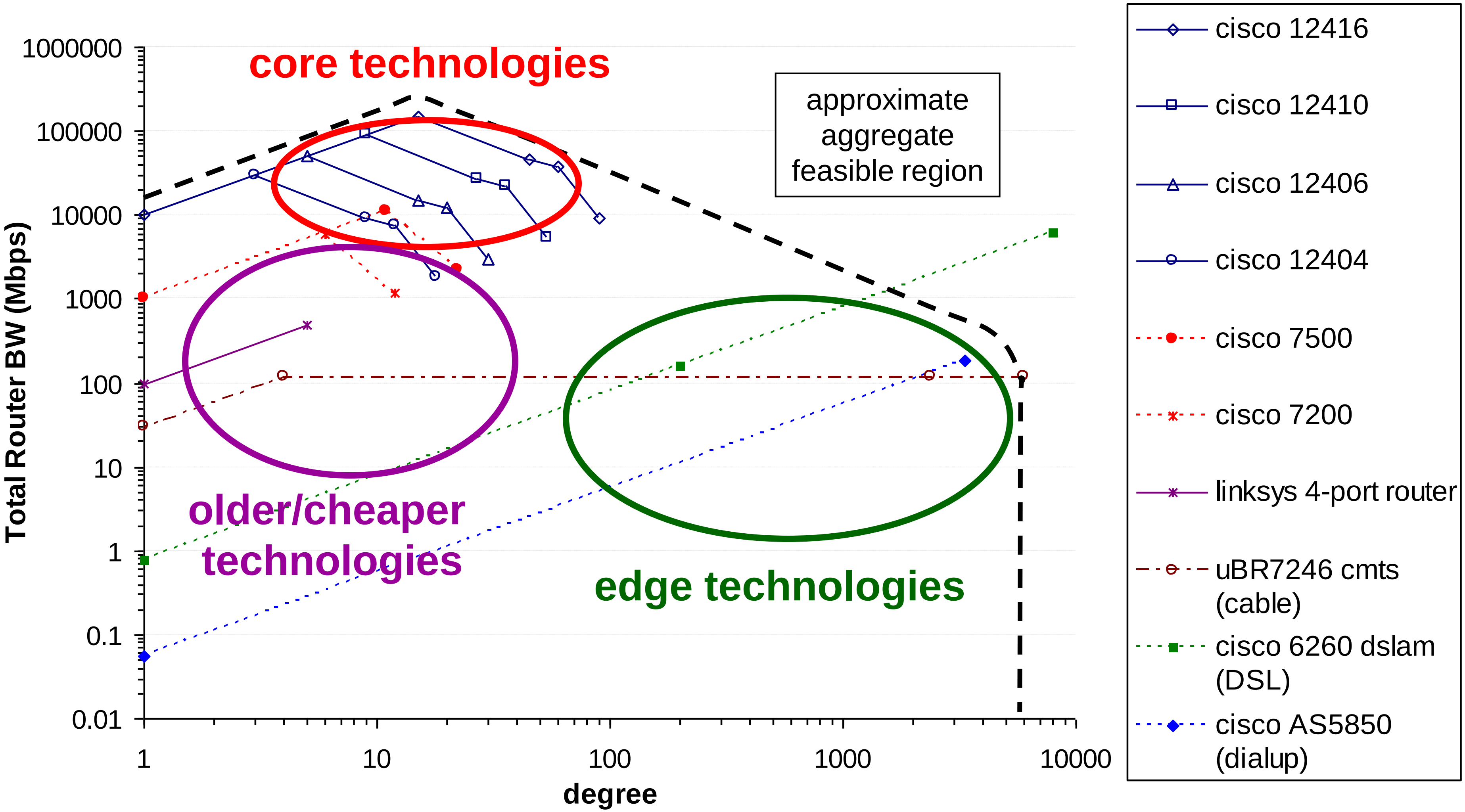
Cisco 12000 Series Routers

Pricing info: State of Washington Master Contract, June 2002

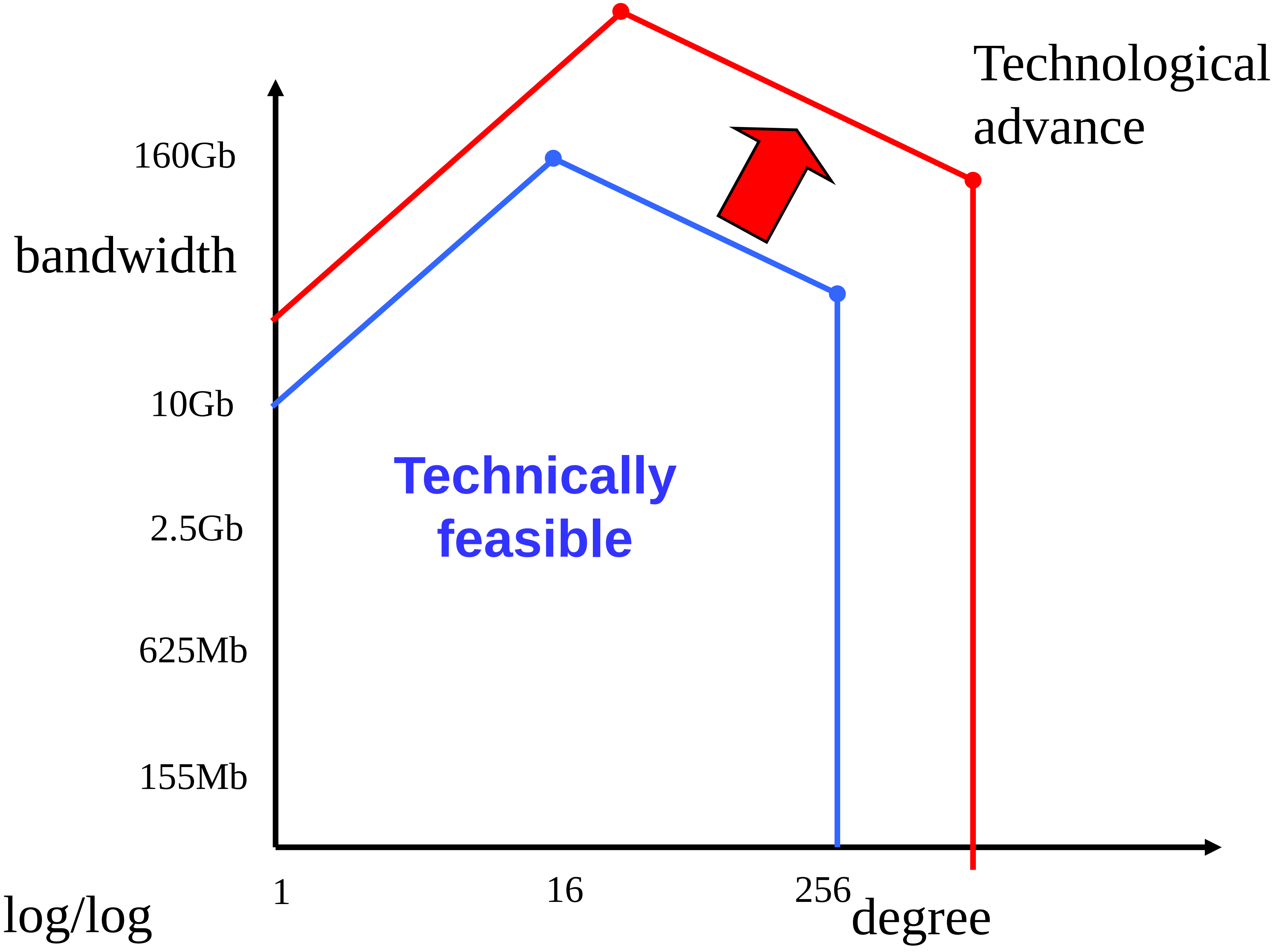
(http://techmall.dis.wa.gov/master_contracts/intranet/routers_switches.asp)

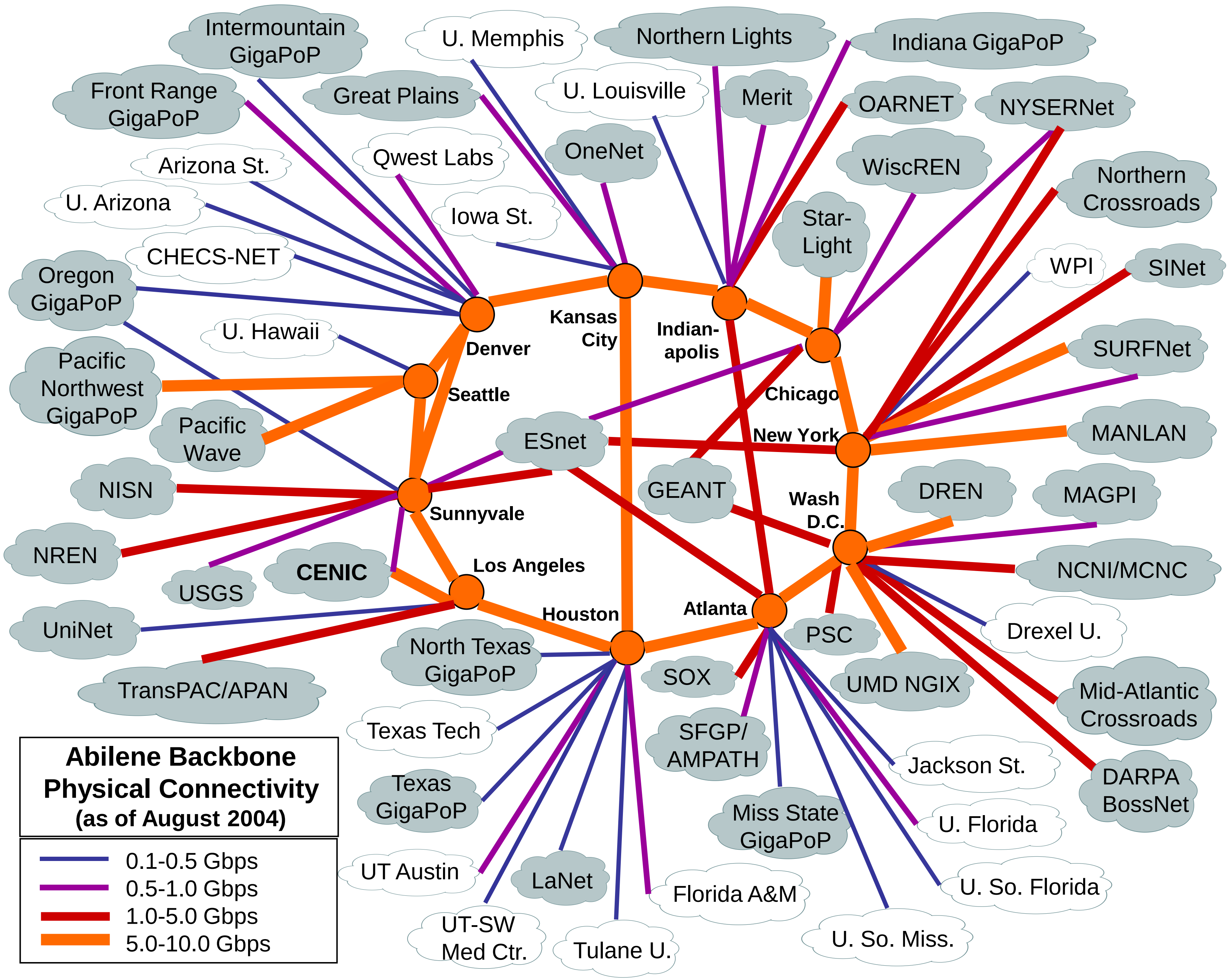


Aggregate Router Feasibility

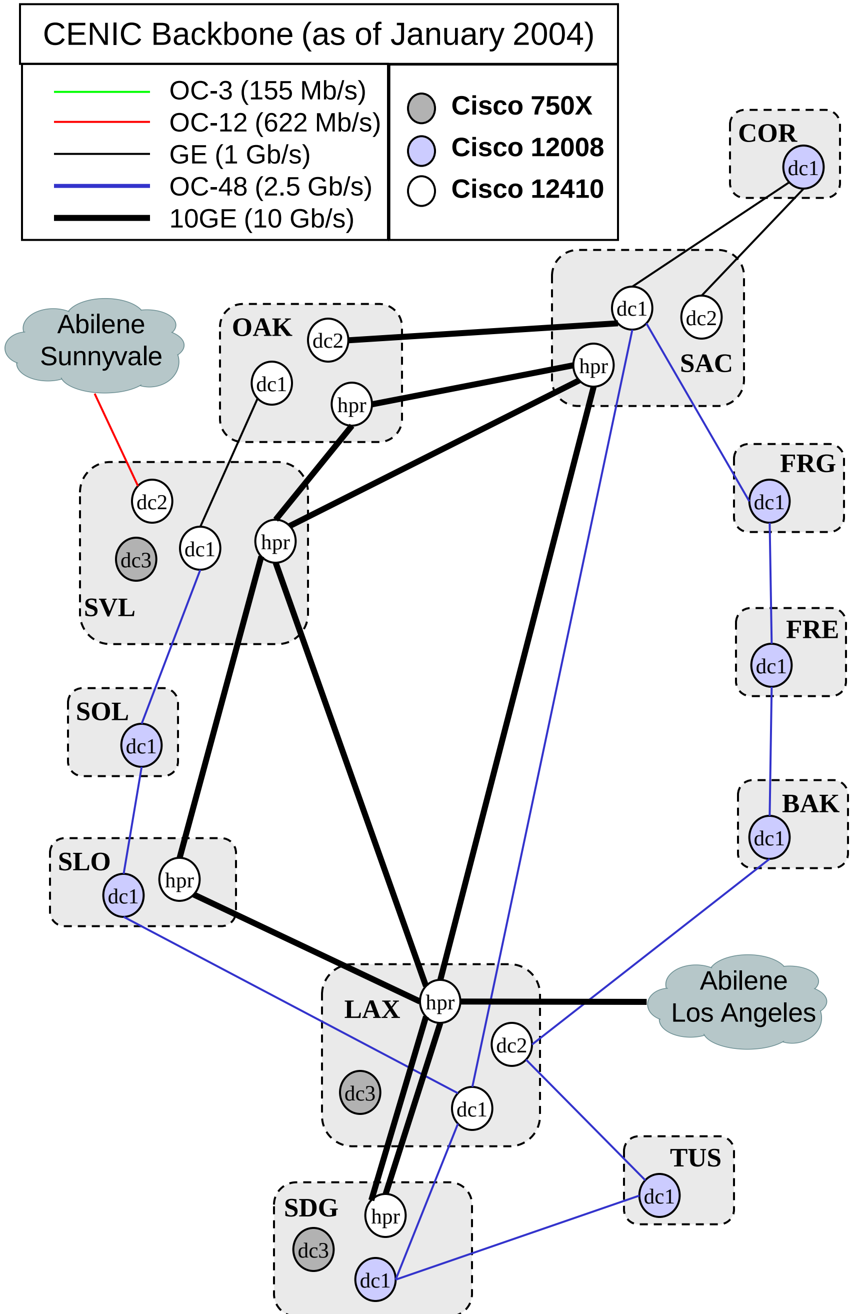


Source: Cisco Product Catalog, June 2002


Technological
advance
160Gb
bandwidth
10Gb
Technically
feasible
2.5Gb
625Mb
155Mb
1
16
256
log/log
degree



Abilene Backbone Physical Connectivity (as of August 2004)

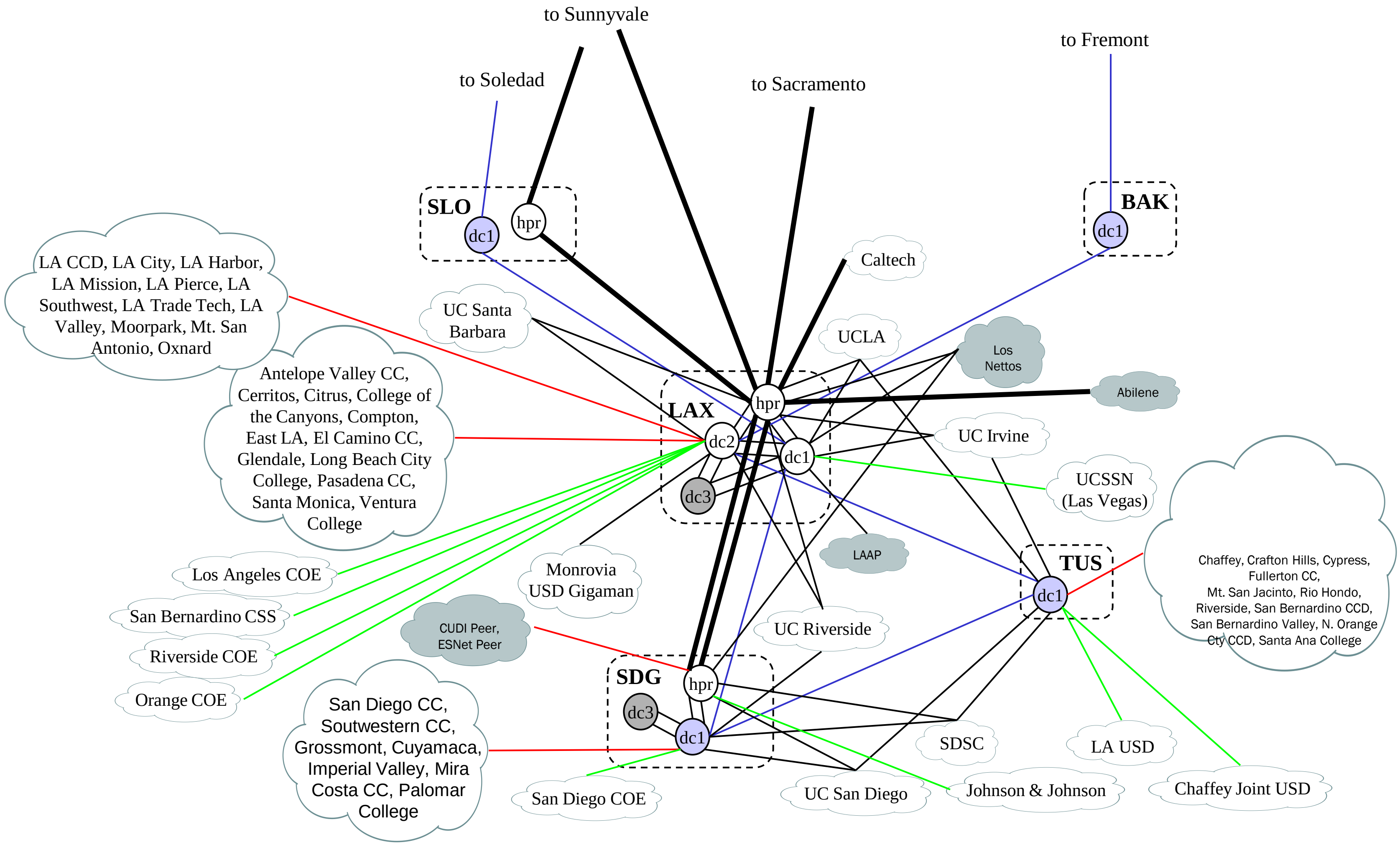


The Corporation for Education Network Initiatives in California (CENIC) acts as ISP for the state's colleges and universities
http://www.cenic.org

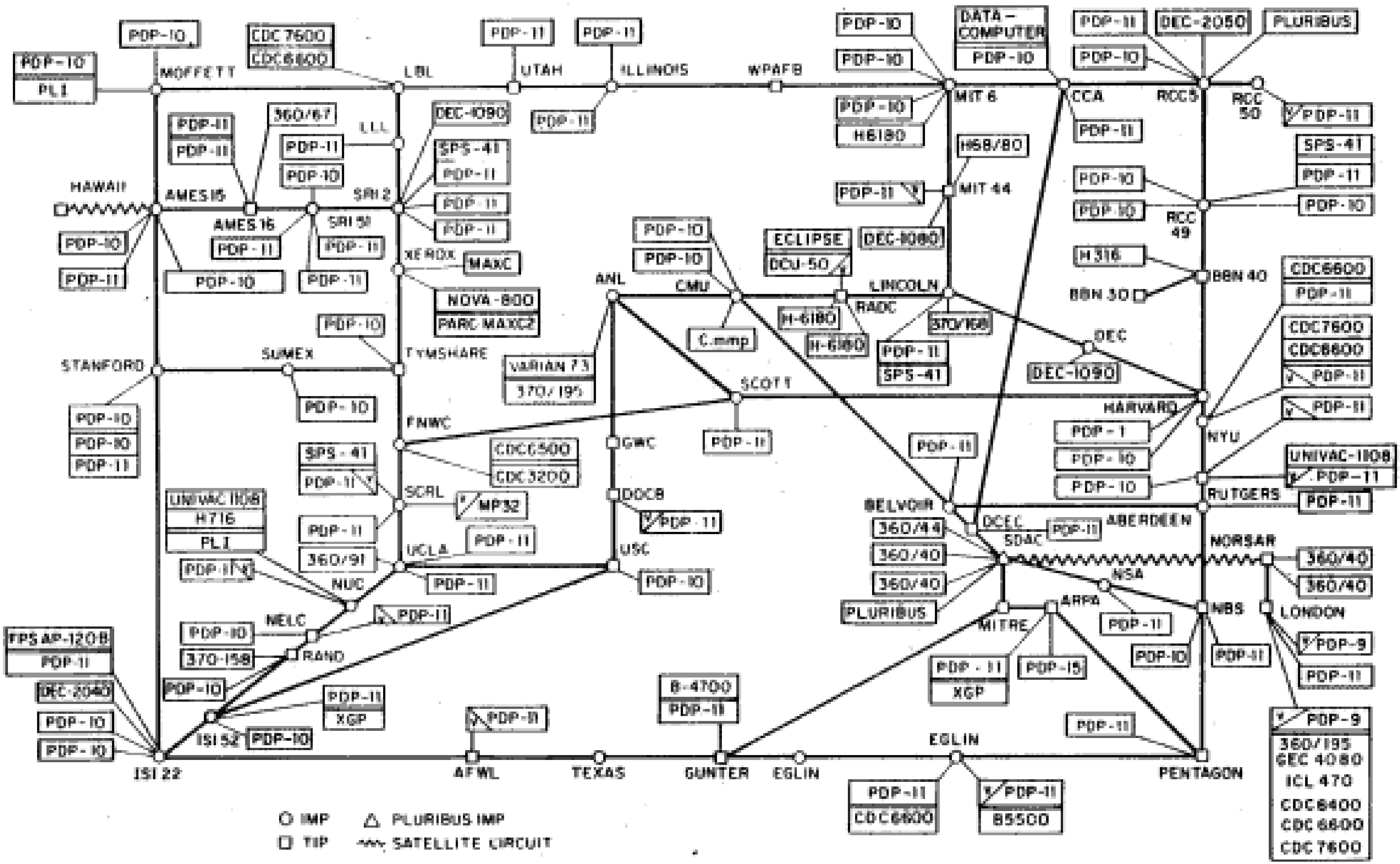
Like Abilene, its backbone is a sparsely-connected mesh, with relatively low connectivity and minimal redundancy.

- no high-degree hubs?
- no Achilles' heel?

CENIC Backbone for Southern California



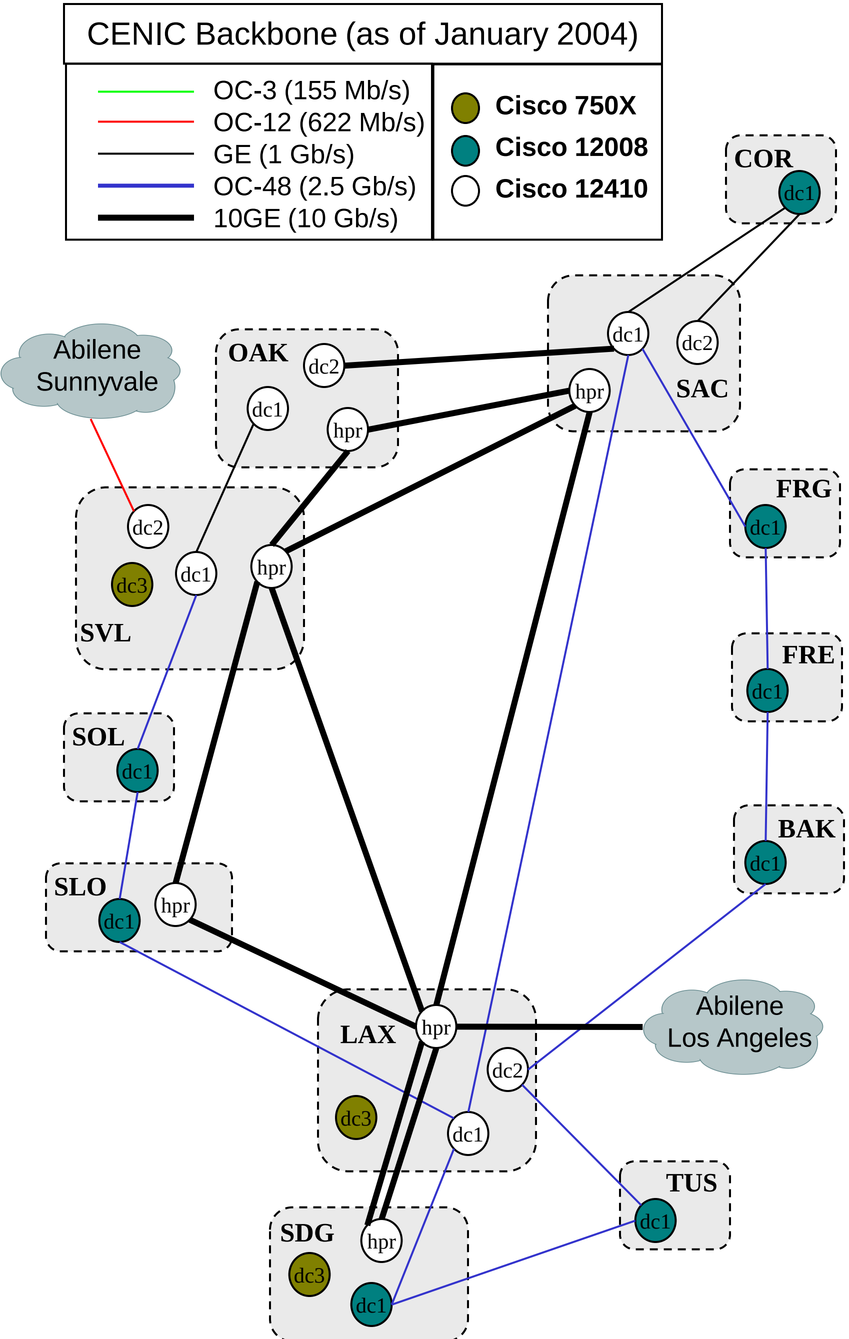
ARPANET LOGICAL MAP, MARCH 1977

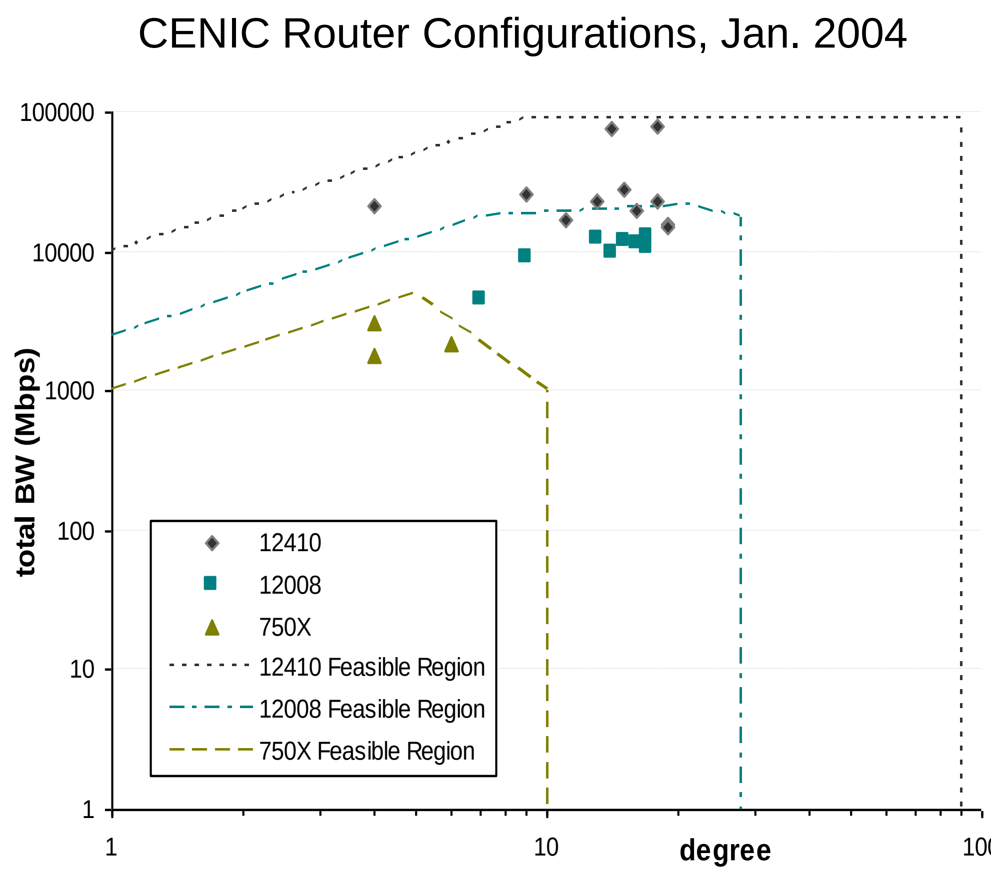


(PLEASE NOTE THAT WHILE THIS MAP SHOWS THE HOST POPULATION OF THE NETWORK ACCORDING TO THE BEST INFORMATION OBTAINABLE, NO CLAIM CAN BE MADE FOR ITS ACCURACY)

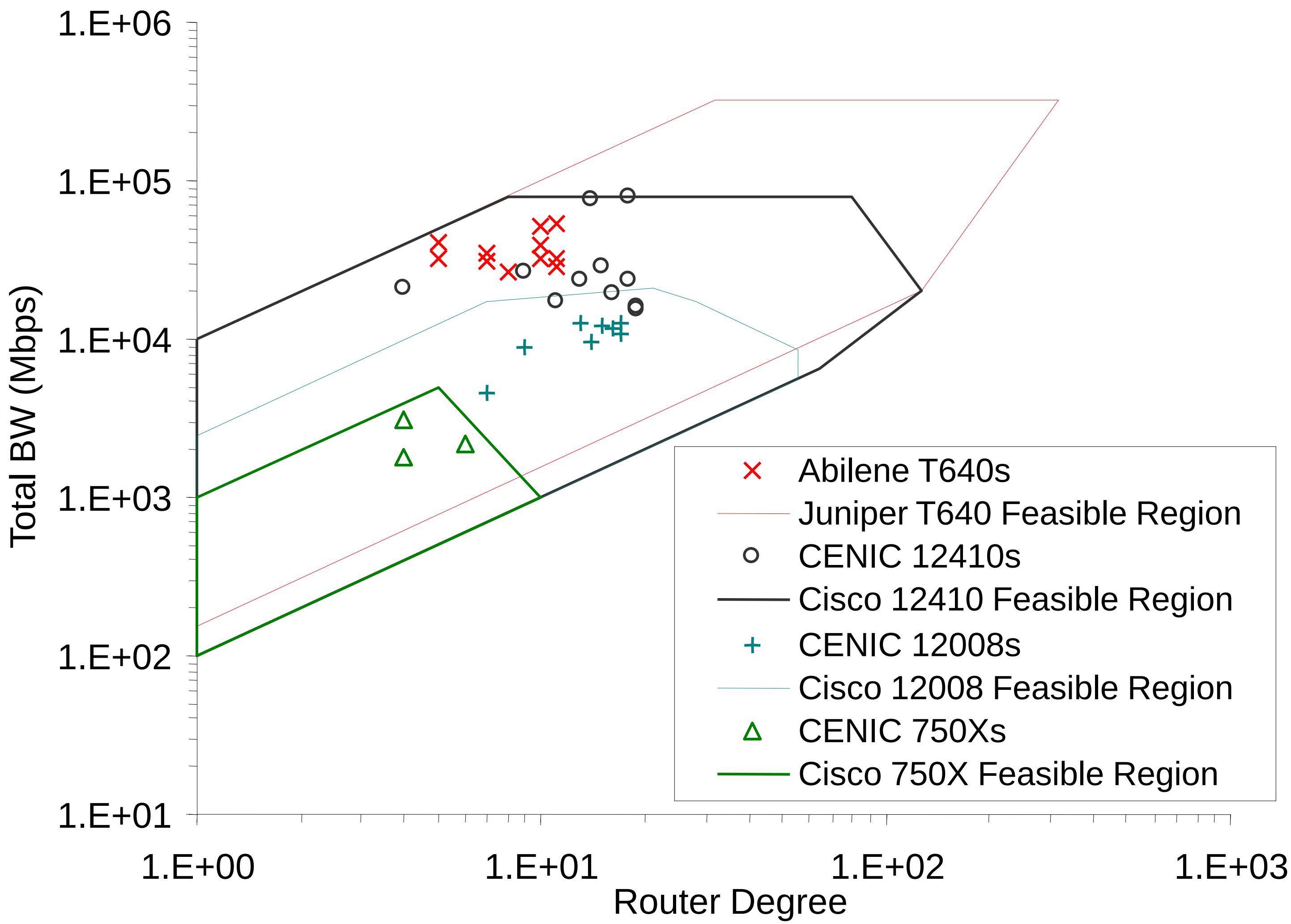
NAMES SHOWN ARE IMP NAMES, NOT (NECESSARILY) HOST NAMES

Corporation for Education Network Initiatives in California (CENIC)

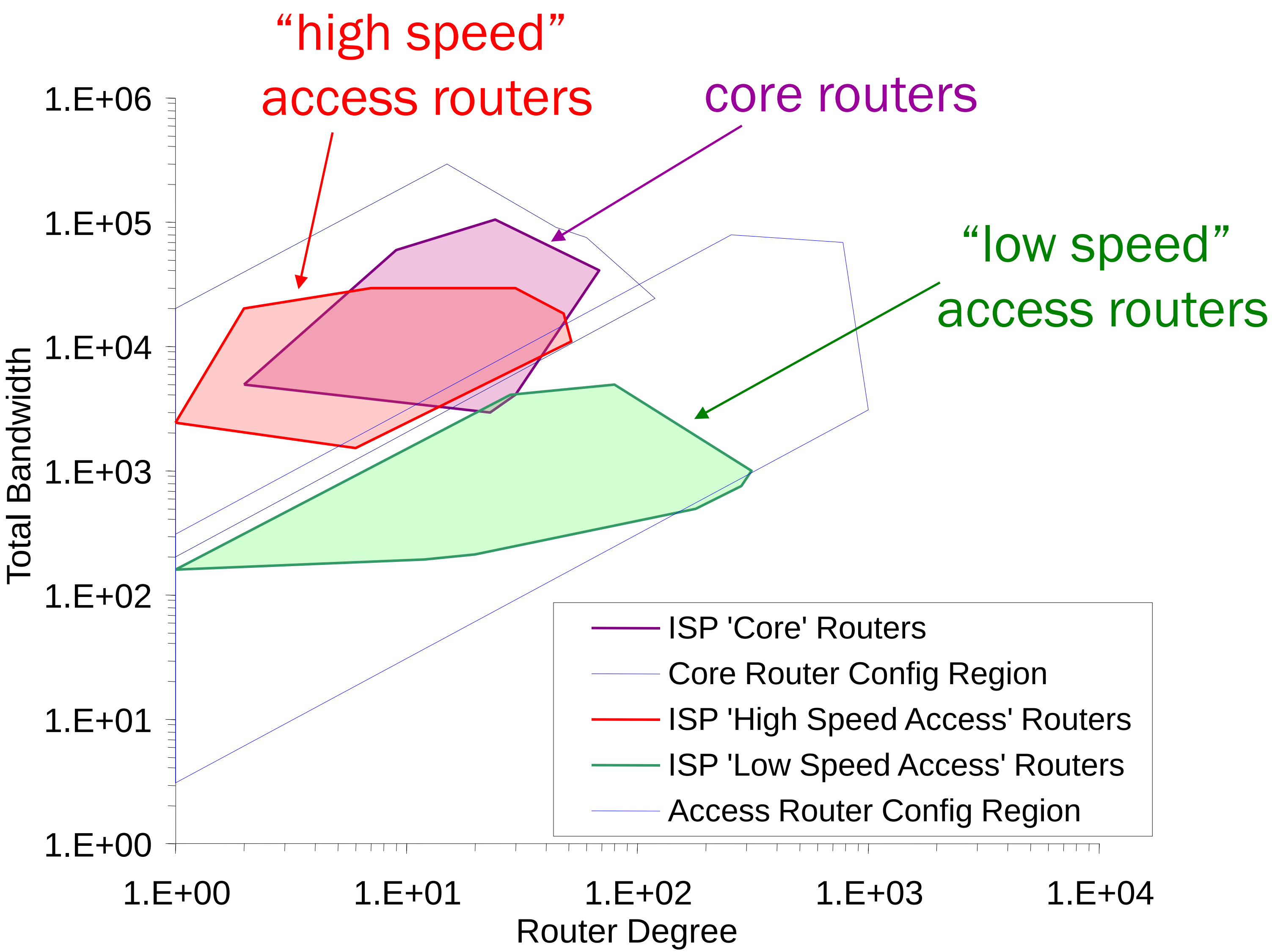




Router Deployment: Abilene and CENIC



AT&T Router Deployment (~2003, courtesy Matt Roughan)



Model Validation: A "hands-on" Approach (cont.)

- Existing router technology
  - Prevents the conclusion of power-law node degrees in the router-level Internet based on available traceroute measurements
  - Allows for the potential of high-degree nodes, but relegates them to the edge of the network
- Examining existing and past real-world networks
  - Real-world router-level topologies look nothing like PA-type networks
  - The results derived from PA-type models of the Internet are not "controversial" – they are simply wrong!
- Bottom line
  - The Internet is exactly the opposite of what scale-free models claim in essentially every meaningful aspect
  - Main reason: random vs. engineered (designed)

Recap: What Network Science says about the Internet

- Power-law (scale-free) node degree distribution
  - (White) lie, damned lie
  - Textbook example of "how to lie with statistics"
- Preferential attachment-type models
  - Damned lie
- Highly popularized claims (e.g., Achilles' heel, fragile/vulnerable to targeted node removal, zero epidemic threshold)
  - Fabrications ...
  - Not "controversial" claims, but simply wrong claims!

Recap: What Network Science says about the Internet (cont.)

- Network Science approach to the Internet
  - Textbook example of how not to do science
  - Dismal analysis of lousy data = bad models
- What is better: Bad models or no models??
  - On the one hand …

    "Bad [models] are potentially important: they can be used to stir up public outrage or fear; they can distort our understanding of our world; and they can lead us to make poor policy choices." (J. Best)
  - … on the other hand …

    Bad models motivate the development of better models ….

The "Math" Perspective of "Network Science and the Internet"

- Starting assumption
  - Node degree distributions follow a power-law
- Rigorous model definition/formulation
  - Preferential attachment-type models
- Rigorous proofs
  - Achilles' heel
  - Fragile/vulnerable to targeted node removal
  - Zero epidemic threshold
- End result is the same
  - The results derived from PA-type models have been made rigorous
  - The application of these results to the Internet are still "lies, damned lies, and statistics".

What Went Wrong?

- No critical assessment of available data
- Ignore all networking-related "details"
  - Randomness enters via generic attachment mechanism
  - Overarching desire to reproduce power law node degree distributions
- Low model validation standards
  - Reproducing observed node degree distribution

How to avoid such Fallacies?

- Know your data!
- Know your statistics!
- Take model validation more serious!
- Apply an engineering perspective to engineered systems!

An Engineering-Centric Perspective

- Must consider the explicit design of the Internet
  - Protocol layers on top of a physical infrastructure
  - Physical infrastructure constrained by technological and economic limitations
  - Emphasis on network performance
  - Critical role of feedback at all levels
- Need to seek models of Internet topology that are explanatory and not merely descriptive.
- Want to consider the ability to match large scale statistics (e.g. power laws) as (at best) secondary evidence of having accounted for key factors affecting design

Internet Modeling: An Engineering Perspective

- Surely, the way an ISP designs its physical infrastructure is not the result of a series of coin tosses …
  - ISPs design their router-level topology for a purpose, namely to carry an expected traffic demand
  - Randomness enters in terms of uncertainty in traffic demands
  - ISPs are constrained in what they can afford to build, operate, and maintain (economics).
  - The “nodes” and “links” are physical things that have hard constraints (technology).
- Decisions of ISPs are driven by objectives (performance) and reflect tradeoffs between what is feasible and what is desirable (heuristic optimization)
  - Constrained optimization as modeling language
- Power laws: Full of sound and fury, signifying nothing!

Key Factors in Network Design

- Technology constraints
  - Router capacity
  - Link capacity
- Economic constraints
  - User demands
  - Link costs
  - Equipment costs
- Performance

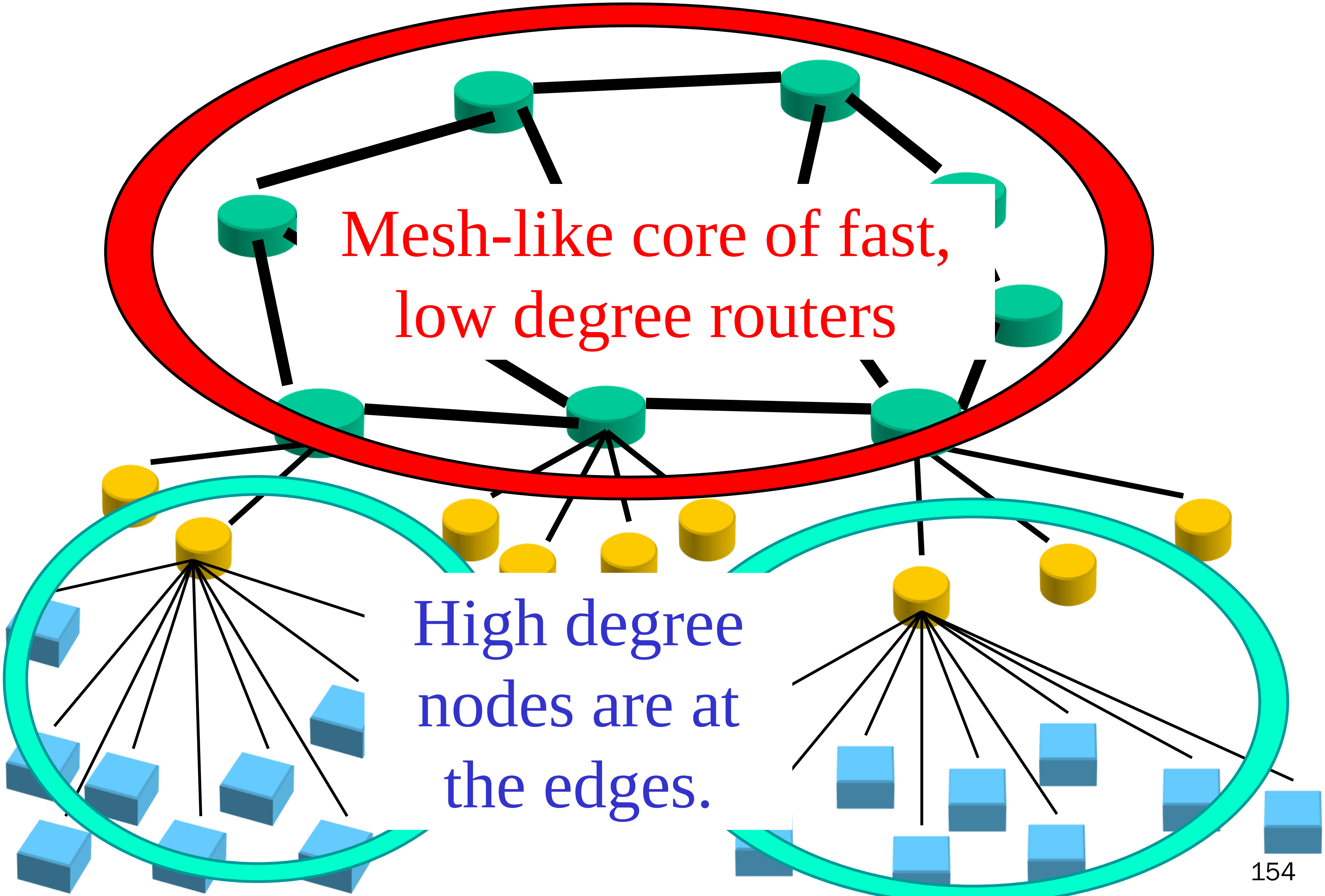
Economic Constraints

- Network operators have a limited budget to construct and maintain their networks
- Links are expensive
- Tremendous drive to operate network so that traffic shares the same links
  - Enabling technology: multiplexing
  - Resulting feature: traffic aggregation at edges
  - Diversity of technologies at network edge (Ethernet, DSL, broadband cable, wireless) is evidence of the drive to provide connectivity and aggregation using many media types

Heuristically Optimal Topologies (HOT)

- Economic considerations alone yield
  - Mesh-like core of high-speed, low degree routers
  - High degree, low-speed nodes at the edge
- Consistent with drivers of router-level network design
  - Technology constraints
  - Link cost (traffic aggregation)
  - End user bandwidth demands
- Consistent with real observed networks
  - Abilene and regional networks
  - Tier-1 ISPs (actual and inferred)

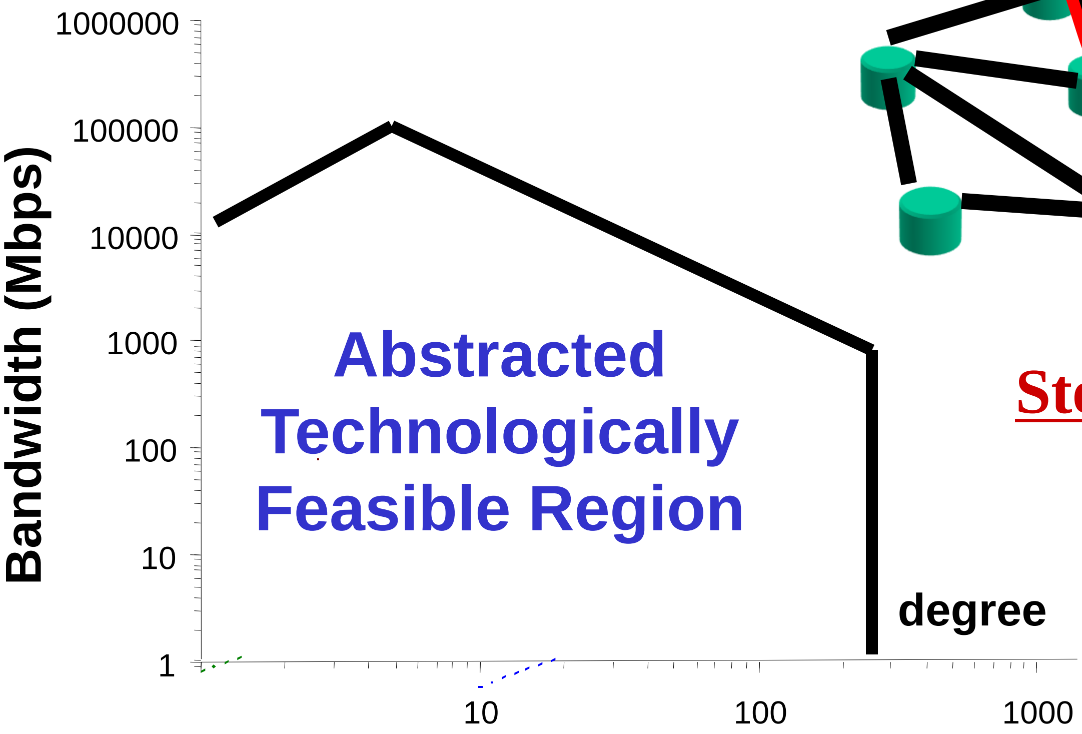
HOT Design Principles



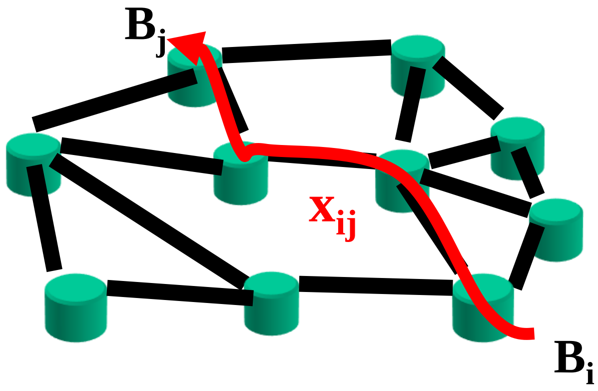
Formalizing the HOT Design

Given realistic technology constraints on routers, how well is the network able to carry traffic?

Step 1: Constrain to be feasible



Step 2: pick traffic demand model



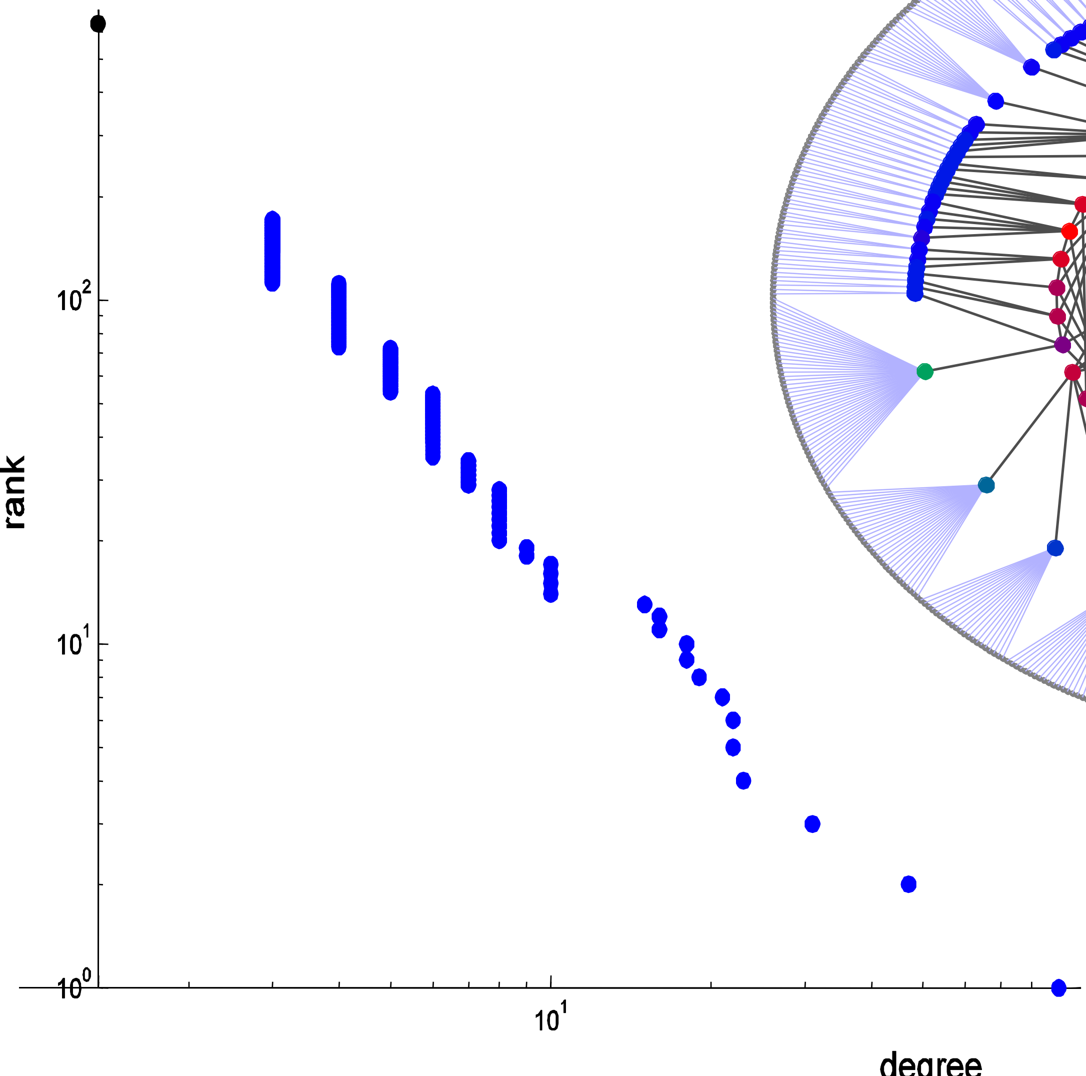
$$x_{ij} \propto B_i B_j$$

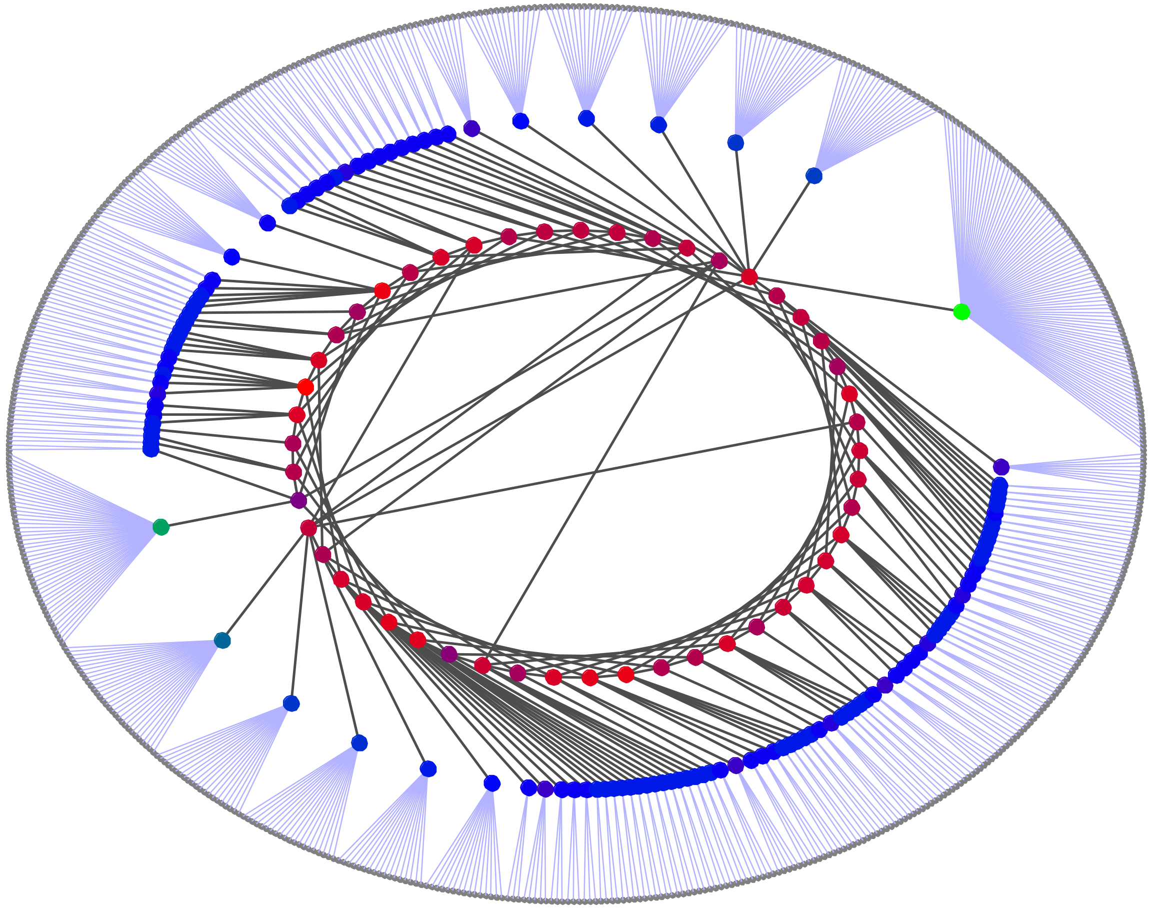
Step 3: Compute max flow

$$\max_{\alpha} \sum_{i,j} x_{ij} = \max \sum_{i,j} \alpha B_i B_j$$

$$s.t. \sum_{i,j:k \in r_{ij}} x_{ij} \le B_k, \forall k$$

HOT Network

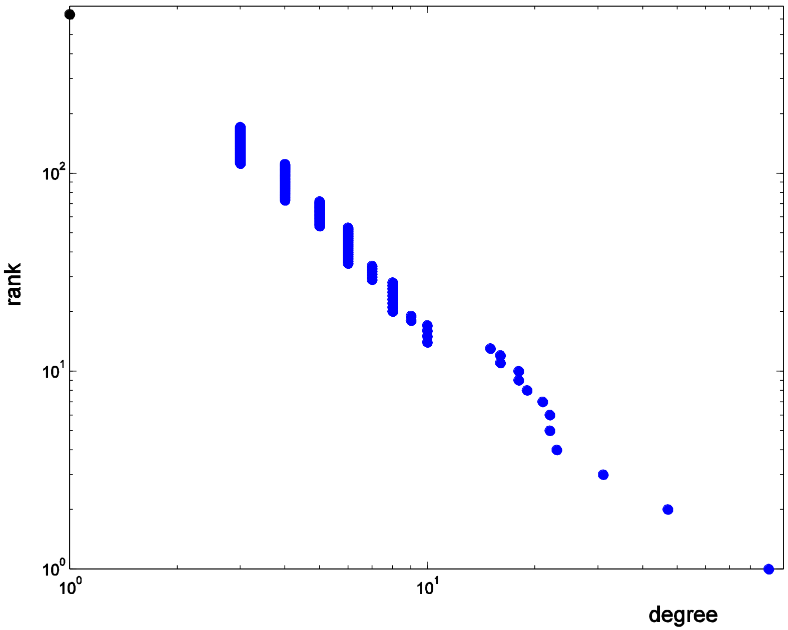




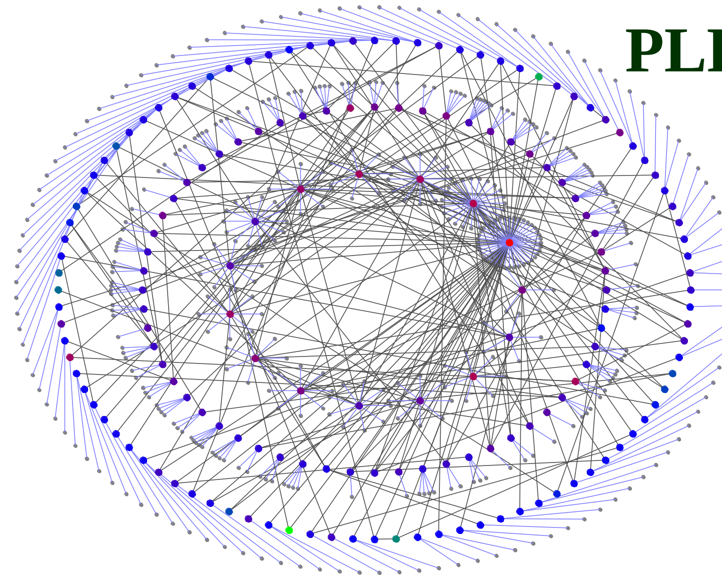
HOT

Highly	**O**ptimized	**T**olerance
Heavily	**O**rganized	**T**radeoffs
Heuristically		

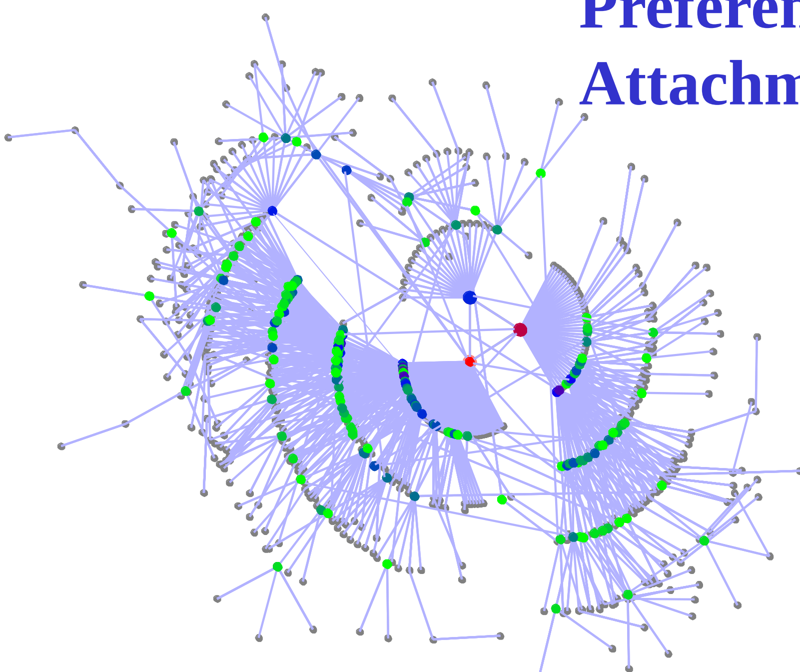
- Based on ideas of Carlson and Doyle
- Complex structure (including power laws) of highly engineered technology (and biological) systems is viewed as the natural by-product of tradeoffs between system-specific objectives and constraints
- Non-generic, highly engineered configurations are extremely unlikely to occur by chance



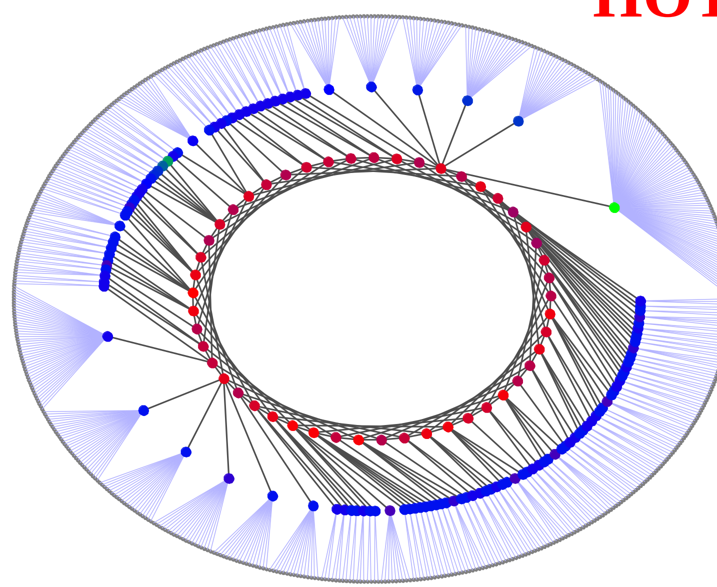
PLRG



Preferential Attachment



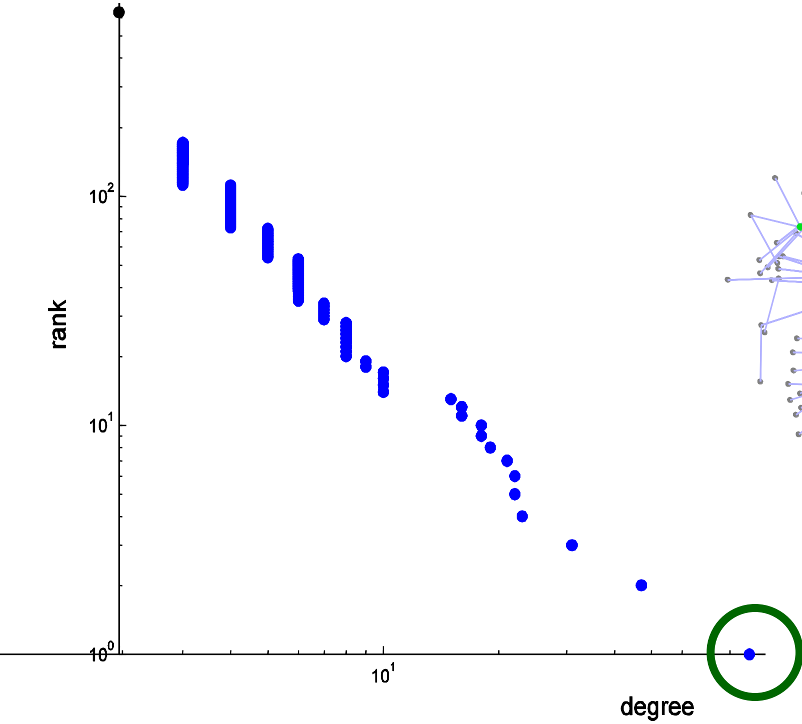
HOT



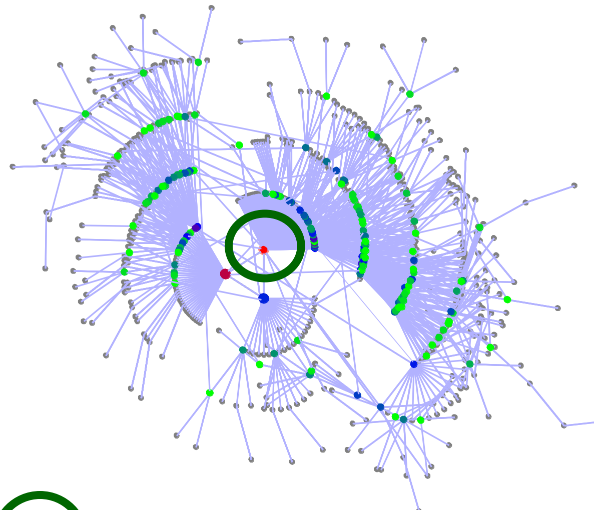
Two Different Perspectives

- Random graph model perspective
  - Randomness enters via coin tosses to determine the presence/absence of links between nodes
  - Match aggregate statistics
  - Suggests high-degree central hubs
- First principles perspective
  - Randomess enters via uncertainty in the environment (i.e., end user traffic demands)
  - Technology and economic constraints
  - Performance
  - Suggest fast, low-degree core routers

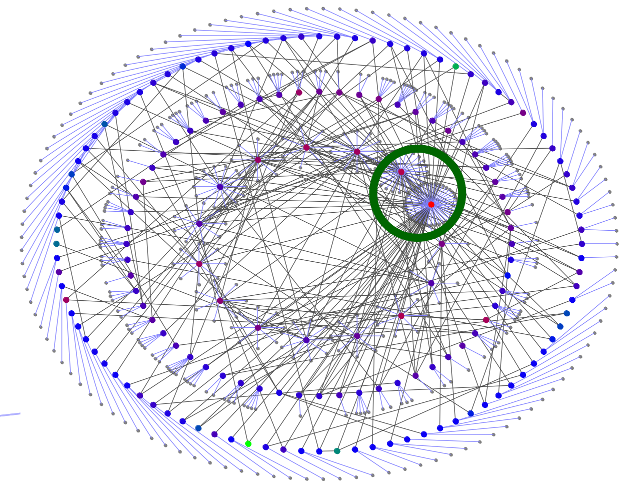
How to reconcile these two perspectives?

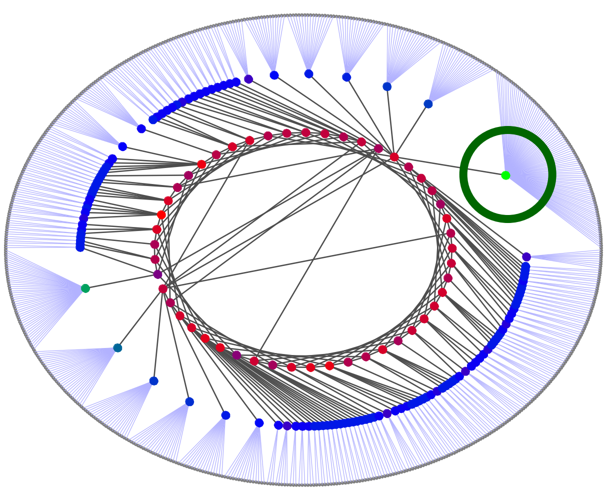


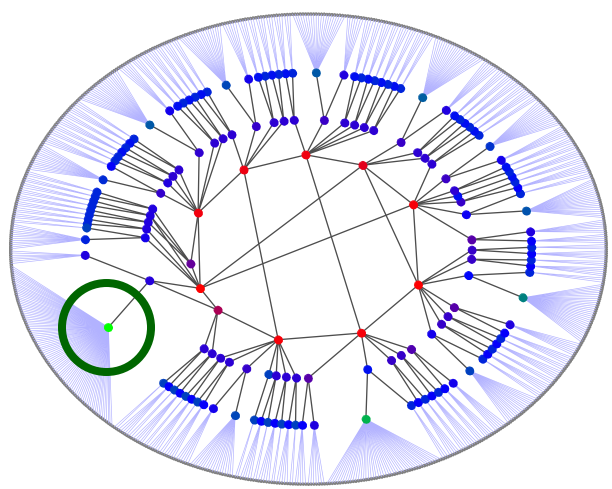
PA

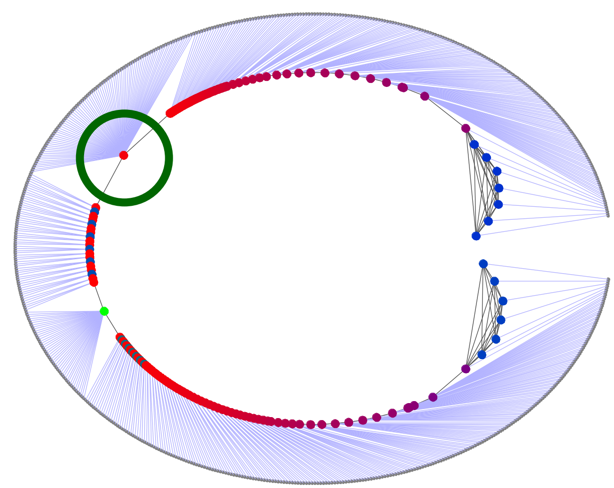


PLRG







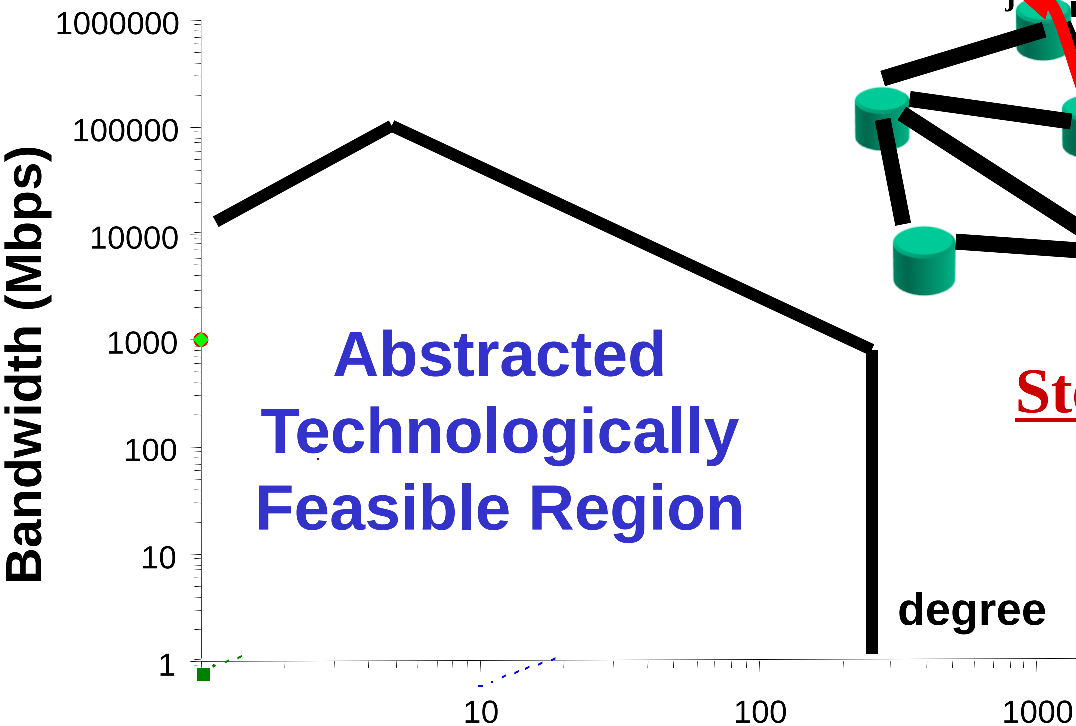


HOT **Abilene-inspired** **Sub-optimal**

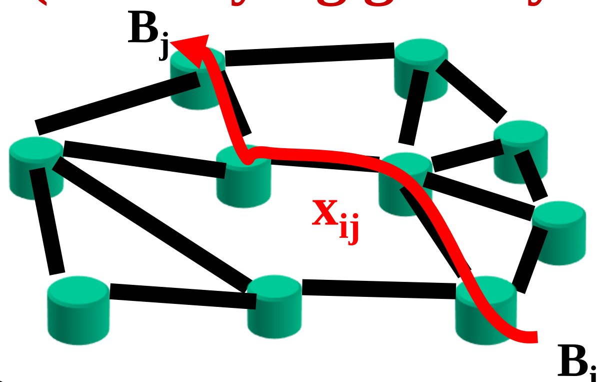
Step 1: Use Internet-Relevant Performance Metric

Given realistic technology constraints on routers and reasonable traffic demands, how well is the network able to carry traffic?

Step 1: Constrain to be feasible



Step 2: Compute traffic demand (underlying gravity model)



$$x_{ij} \propto B_i B_j$$

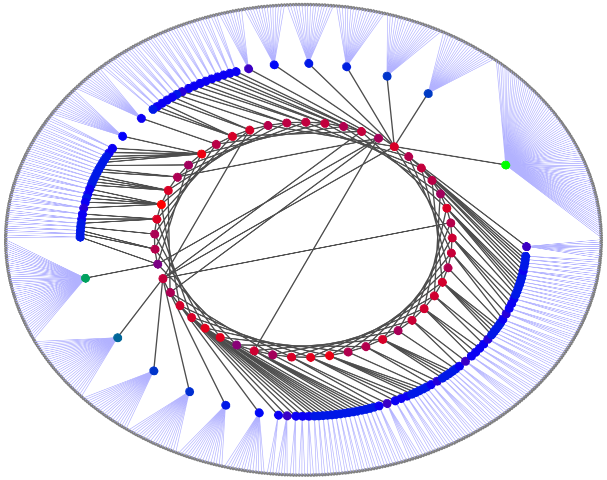
Step 3: Compute max flow

$$\max_{\alpha} \sum_{i,j} x_{ij} = \max \sum_{i,j} \alpha B_i B_j$$

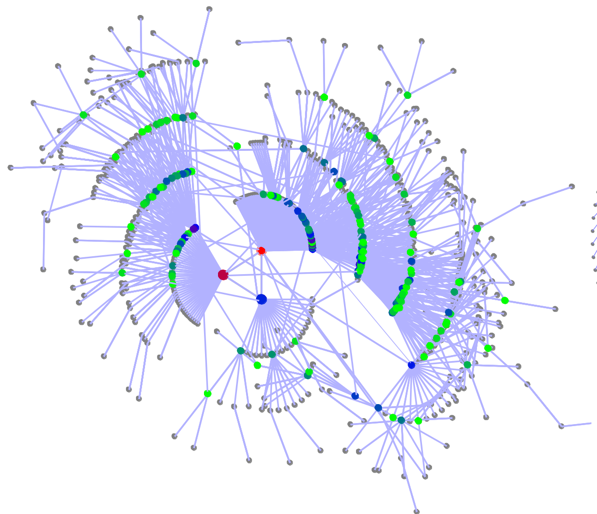
$$s.t. \sum_{i,j:k \in r_{ij}} x_{ij} \le B_k, \forall k$$

Structure Determines Performance

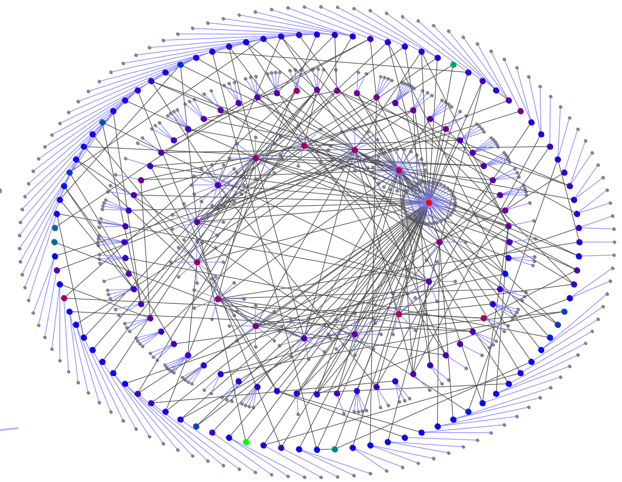
HOT



PA



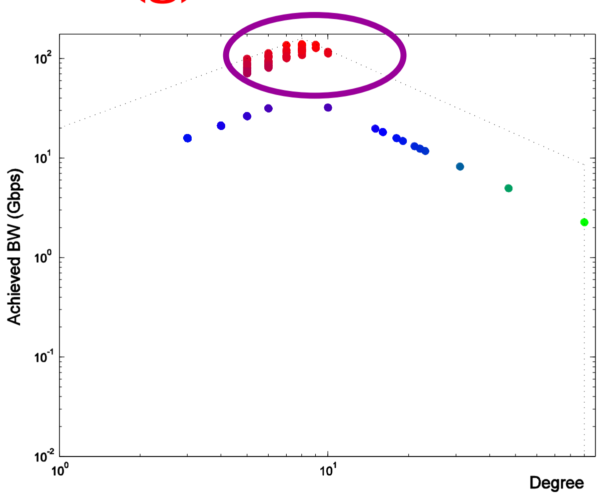
PLRG

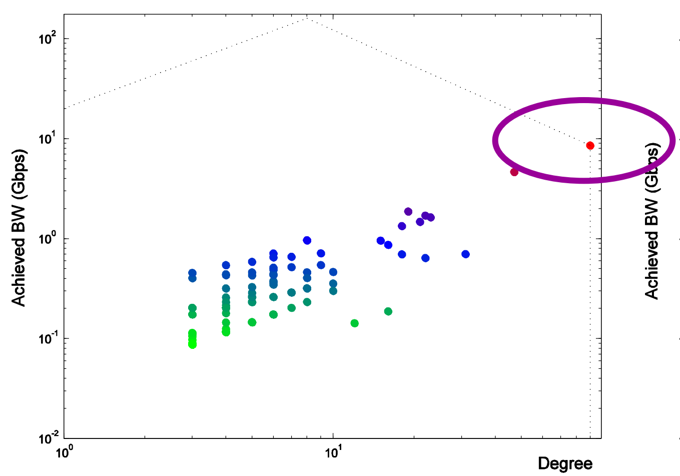


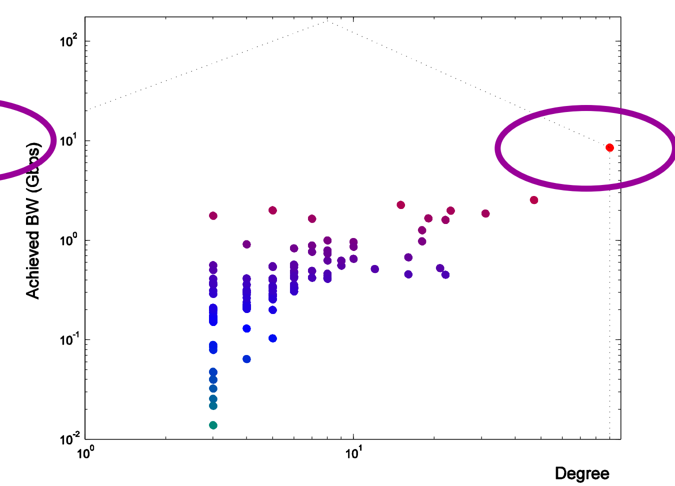
$P(g) = 1.13 \times 10^{12}$

$P(g) = 1.19 \times 10^{10}$

$P(g) = 1.64 \times 10^{10}$







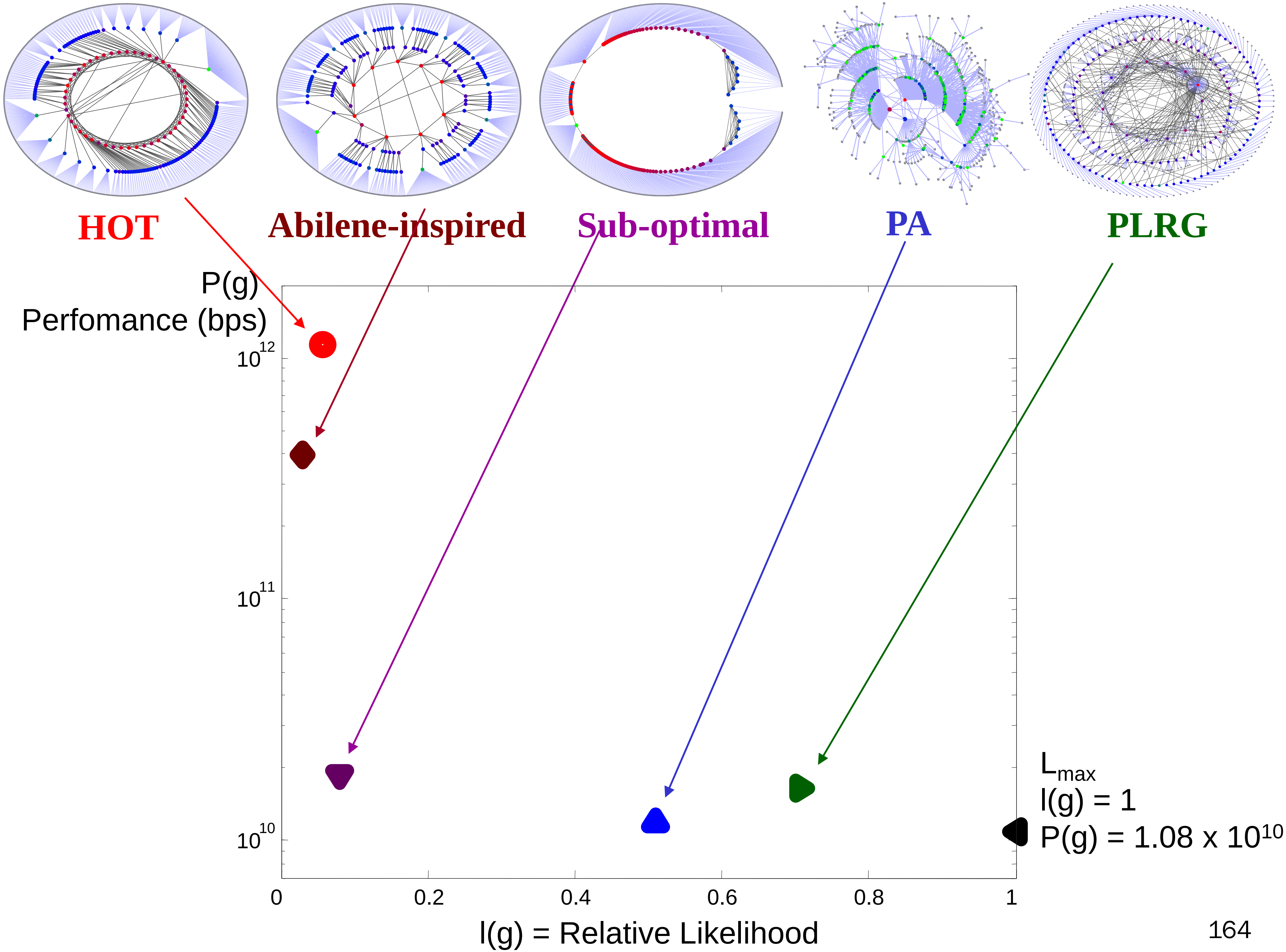
Step 2: Use Graph-Theoretic Metric

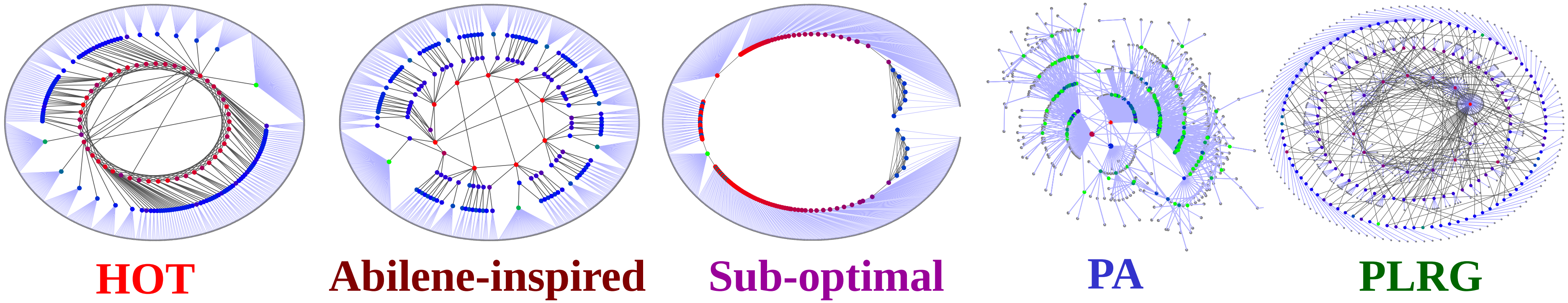
Define the metric $L(g) = \sum_{\substack{i,j \\ connected}} d_i d_j$ (d_i = degree of node i)

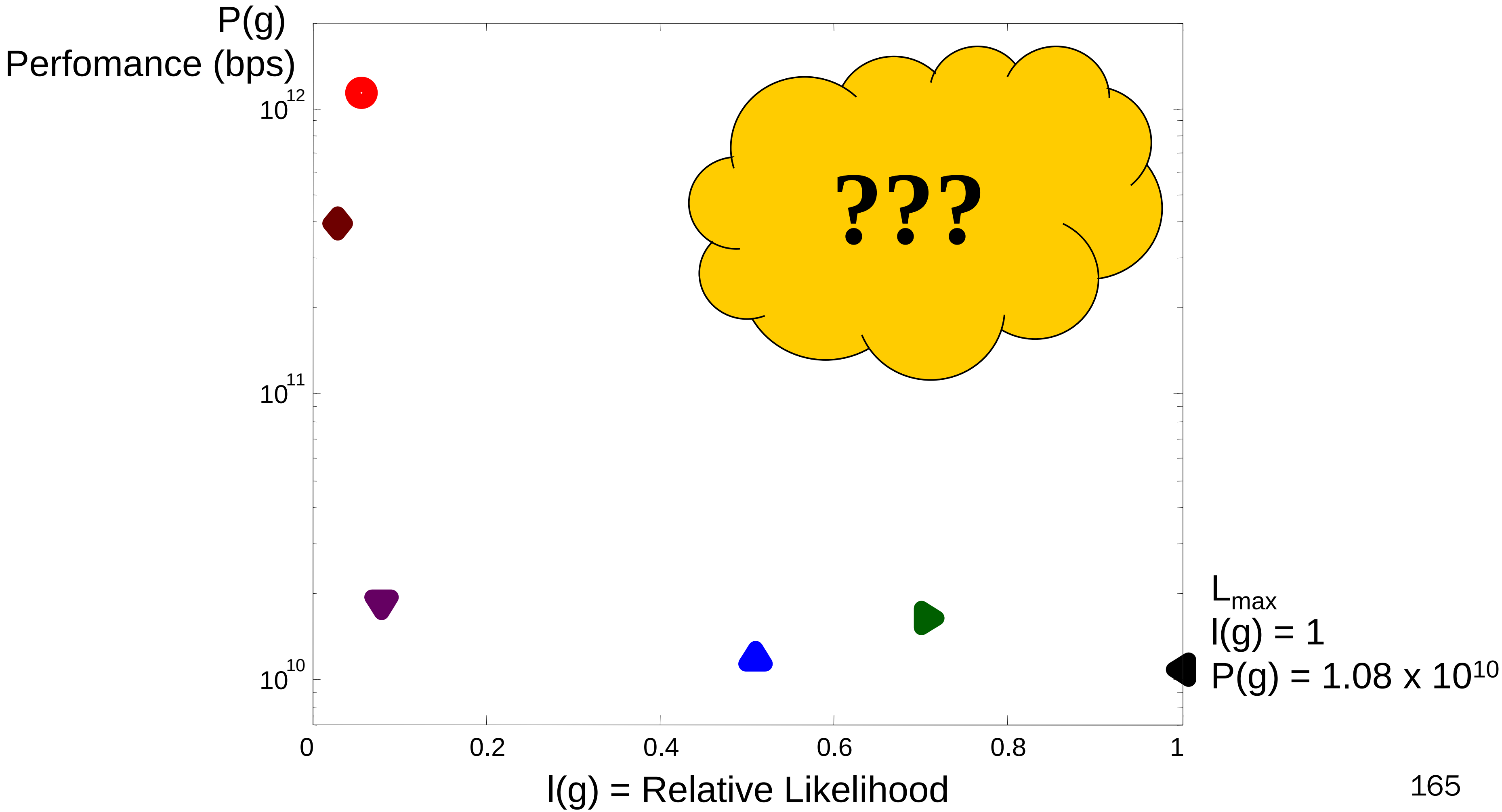
- Easily computed for any graph
- Depends on the structure of the graph, not the generation mechanism
- L(g) is large: connect high-degree nodes

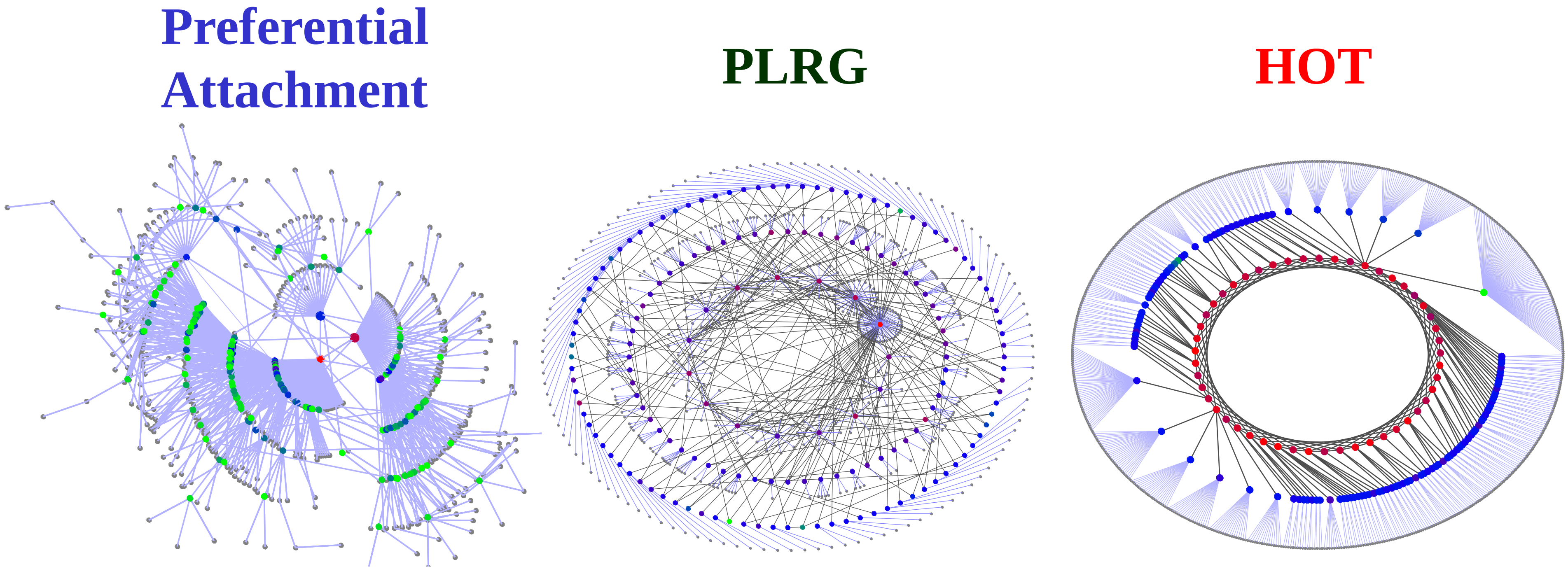
Interpretations

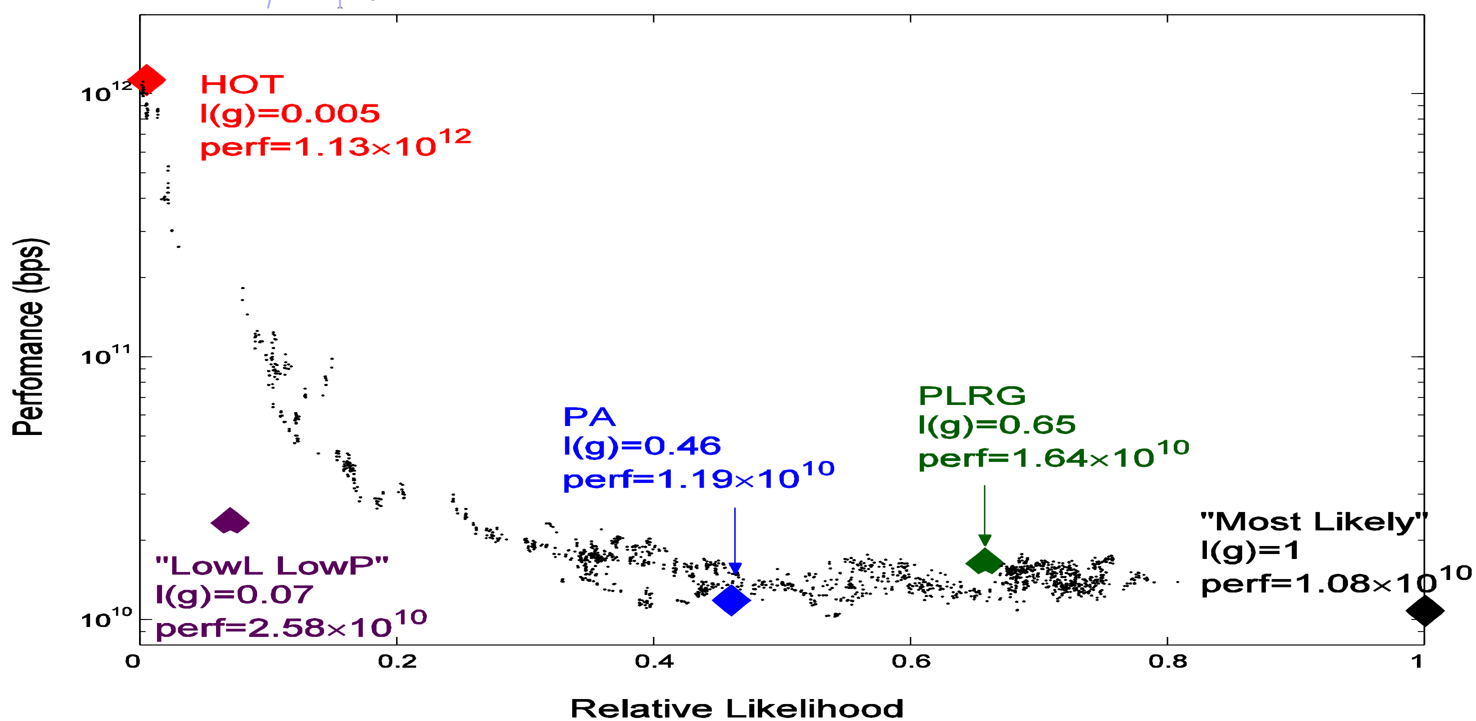
- $L(g) \propto$ LogLikelihood (LLH) of *g* (random graph models)
- *L*(*g*) measures the extent to which *g* has "hub-like" core
- *L*(*g*) measures the extent to which *g* is "scale-free"
- *L*(*g*) measures the extent to which *g* is "self-similar'


HOT
Abilene-inspired
Sub-optimal
PA
PLRG
P(g)
Perfomance (bps)
10^{12}
10^{11}
10^{10}
0
0.2
0.4
0.6
0.8
1
L_{max}
l(g) = 1
P(g) = 1.08 x 10^{10}
l(g) = Relative Likelihood


HOT
Abilene-inspired
Sub-optimal
PA
PLRG


P(g)
Perfomance (bps)
10^12
10^11
10^10
???
L_max
l(g) = 1
P(g) = 1.08 x 10^10
0
0.2
0.4
0.6
0.8
1
l(g) = Relative Likelihood

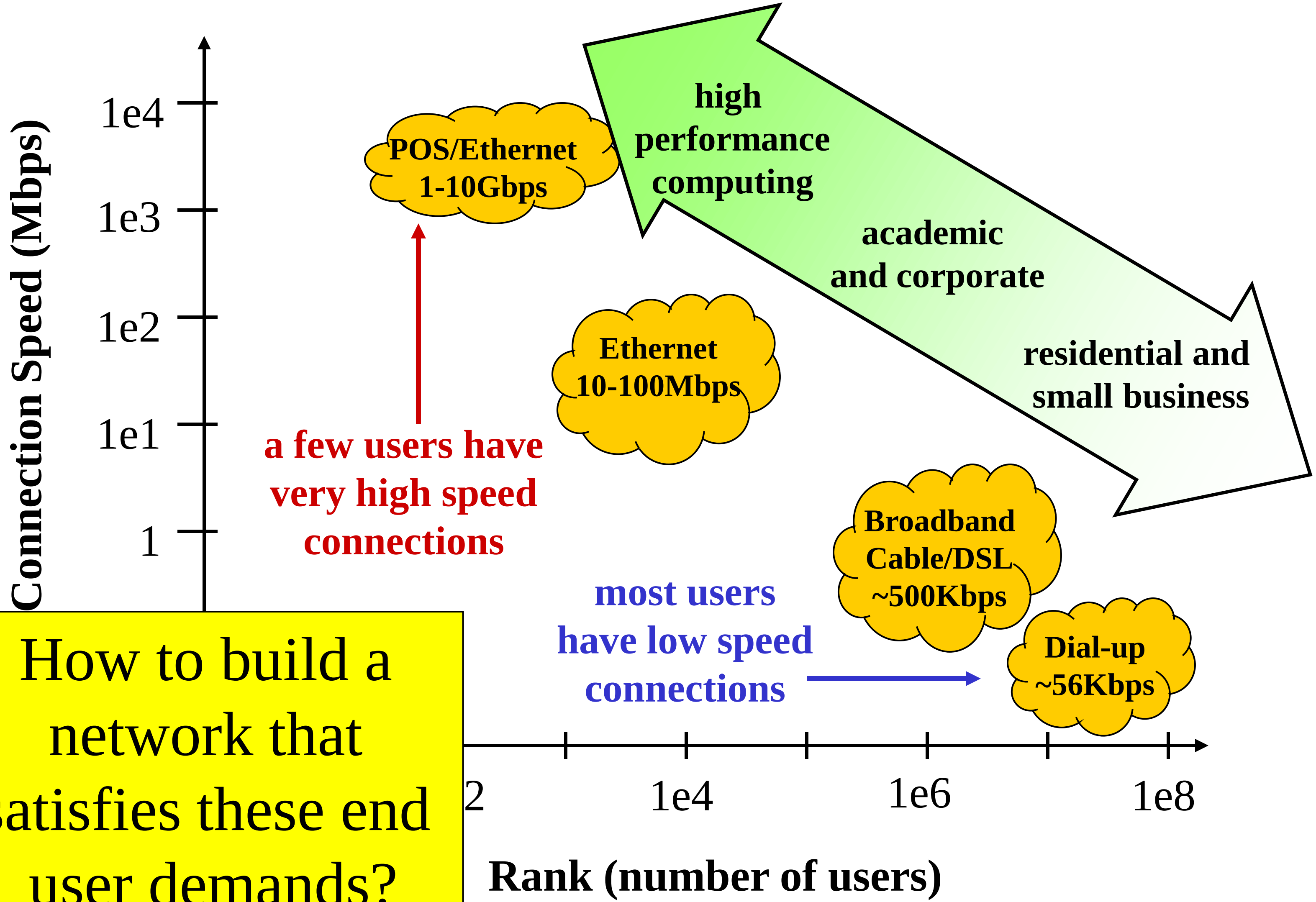

Preferential Attachment
PLRG
HOT


HOT
l(g)=0.005
perf=1.13×10^12
"LowL LowP"
l(g)=0.07
perf=2.58×10^10
PA
l(g)=0.46
perf=1.19×10^10
PLRG
l(g)=0.65
perf=1.64×10^10
"Most Likely"
l(g)=1
perf=1.08×10^10
Perfomance (bps)
Relative Likelihood
10^12
10^11
10^10
0
0.2
0.4
0.6
0.8
1

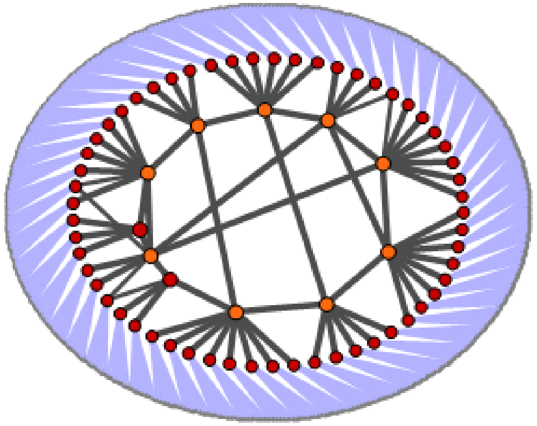
What about High Variability of Node Degrees?

- Same degree distribution can have different core structures
  - PA, PLRG, HOT, …
- Same core structure can have different degree distributions
  - Uniform low-end user bandwidth demands
  - Uniform high -end user bandwidth demands
  - Highly variable-end user bandwidth demands

Internet End-User Bandwidths



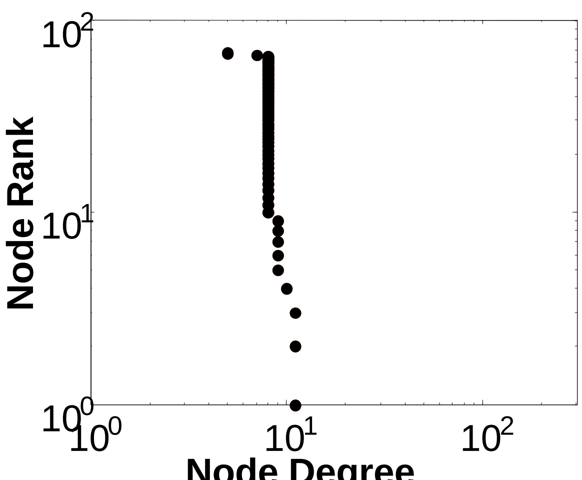
How to build a network that satisfies these end user demands?

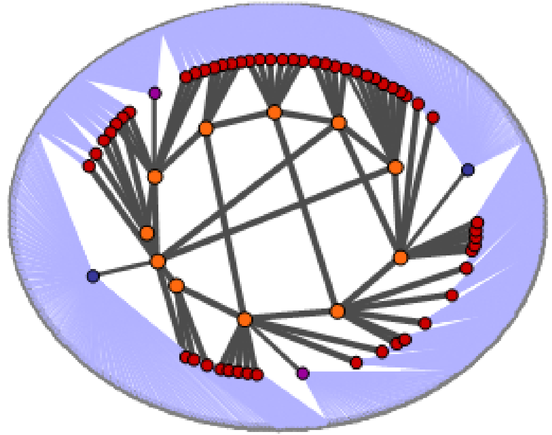


Abilene-inspired core

Uniform high BW users

Low variability deg dist.

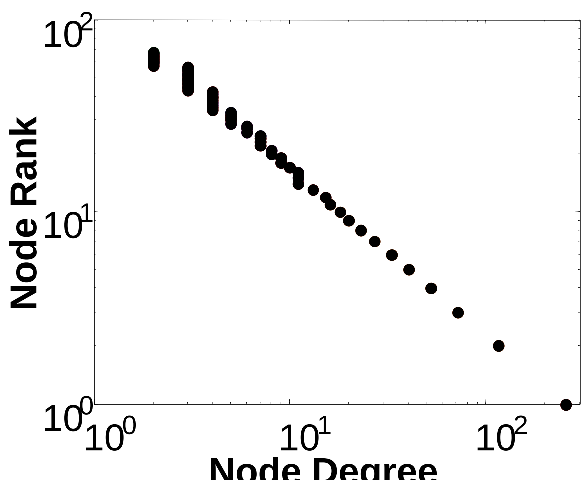


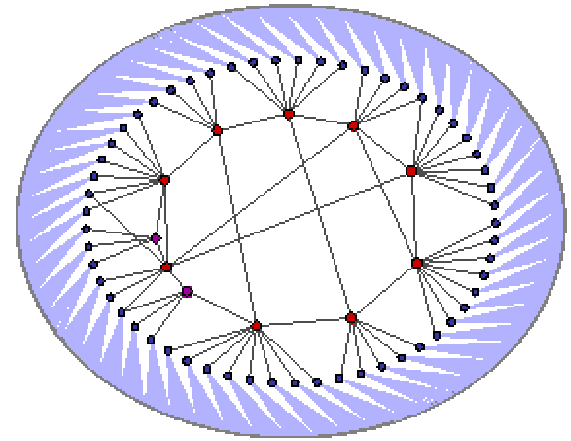


Abilene-inspired core

High variability at edges

Power-law deg distribution

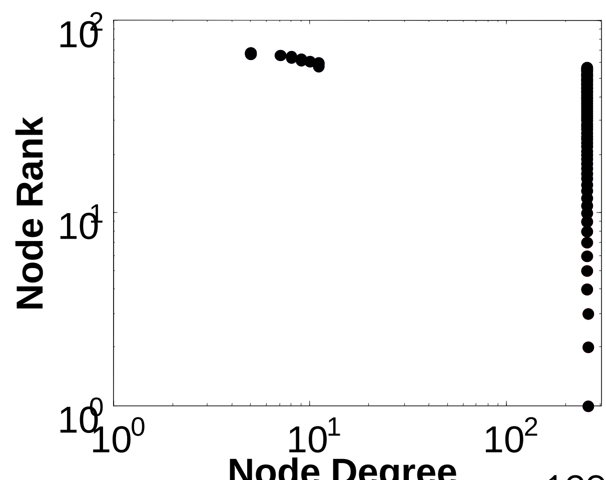




Abilene-inspired core

Uniform low BW users

low variability deg dist.



What about High Variability of Node Degrees?

- Same degree distribution can have different core structures
  - PA, PLRG, HOT, …
- Same core structure can have different degree distributions
  - Uniform low-end user bandwidth demands
  - Uniform high -end user bandwidth demands
  - Highly variable-end user bandwidth demands

- **Root cause of high variability in node degrees**
  - End user bandwidth demands
- **So much for power-laws!**
  - Full of sound and fury, signifying nothing!

Comparing HOT and SF

HOT networks, Internet

- Core: Mesh-like, low degree
- Edge: High degree
- Robust to random failures
- Robust to "attack"

SF networks

- Core: Hub-like, high degree
- Edge: Low degree
- Robust to random failures
- Fragile to "attack"

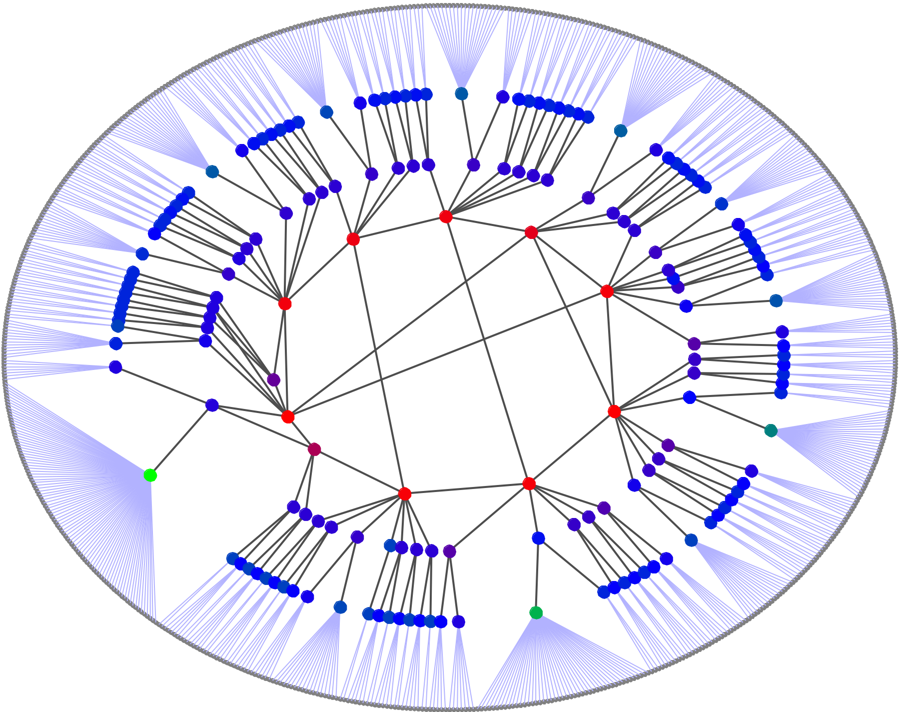
+ objectives and constraints

HOT networks, Internet:

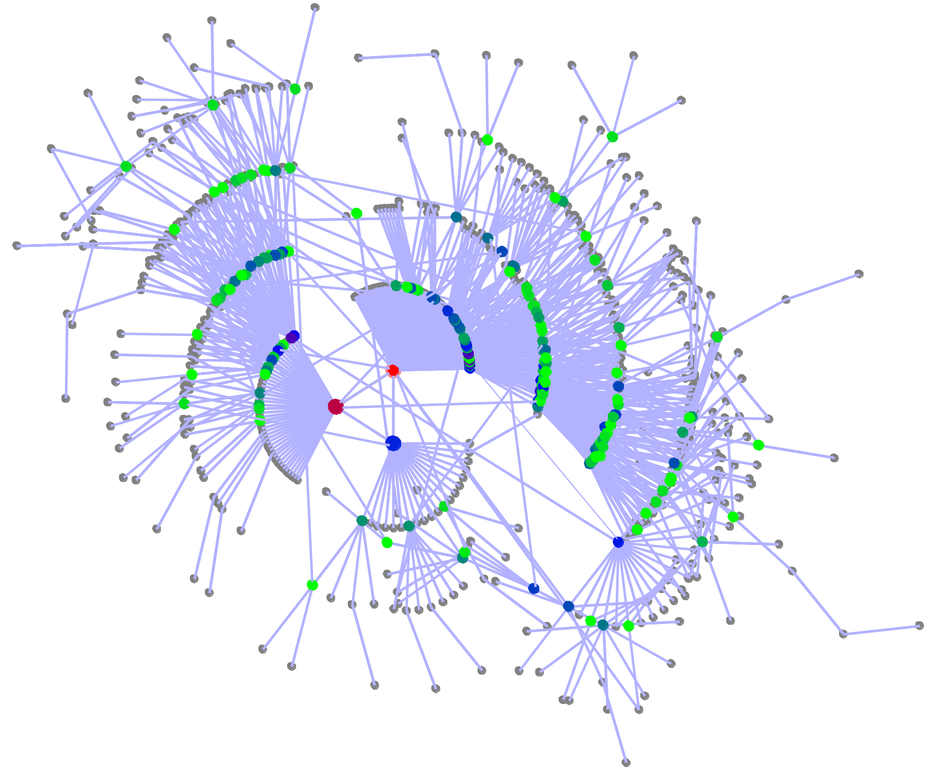
- High performance
- Low link costs
- Unlikely, rare, designed
- Destroyed by rewiring

SF networks:

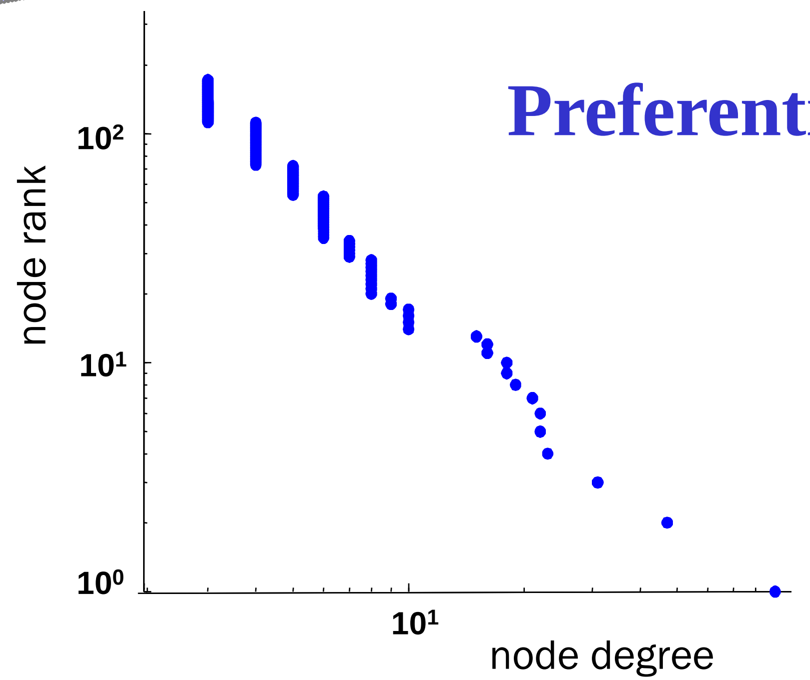
- Low performance
- High link costs
- Highly likely, generic
- Preserved by rewiring



HOT model



Preferential Attachment



HOT- vs. PA-type Network Models

Features	HOT-type/ Internet	PA-type models
Core nodes	Fast, low degree	Slow, high degree
High degree nodes	Edge	Core
Degree distribution	Highly Variable	Power law
Generation	Designed	Random
Performance	High throughput	Low throughput
Attack Tolerance	Robust	Fragile
Fragility	Hijack network	Attack hubs

HOT-type Network Models

- Important lessons learned
  - Know your data! – they typically reflect what we can measure rather than what we would like to measure
  - Avoid the allure of PA-type network models! – there exist more relevant, interesting, and rewarding network models that await discovery
  - Details do matter! – beware of layers, protocols, feedback, ...
- Key features of HOT models
  - Consistent with existing ISP router-level topologies
  - Consistent with existing technologies
  - Consistent with engineering principles
  - Consistent with (complementary) measurements
  - Node degree distribution is a non-issue

HOT-type Network Models (cont.)

- Recent alternatives to PA-type models
  - Motivated by the failure of the PA-type models
  - Extremely unlikely to occur at random
- New paradigm for network modeling
  - Network modeling $\neq$ exercise in data fitting
  - Network modeling = exercise in reverse-engineering
  - Constrained optimization as modeling language
- Constrained optimization as mathematical modeling language
  - Optimization of tradeoffs between multiple functional objectives of networks
  - Subject to constraints on their components
  - With an explicit source of uncertainty (in the environment) against which solutions must be tolerant or robust

Litmus Test for Newly Proposed Network Models

- Make node degree distribution a non-issue
  - Good reasons
    - High-quality data but low variability (e.g., exponential)
    - Low-quality data
    - High-quality data and high variability (e.g., power-laws)
  - PA-type models
    - DOA -- dead on arrival
  - Only reasonable alternative
    - Bring in and rely on domain knowledge
- What new kinds of measurements does the proposed model suggest for the purpose of model validation
  - PA-type models: none
  - HOT models: get data on existing router technology

Implications of this Engineering Perspective

- Dynamics of graphs
  - Evolution of connectivity structures
  - Evolution of (internal) node/link structure
- Dynamics over graphs
  - Traffic dynamics/matrix (bytes, packets, flows, …)
- Challenging feedback problem
  - Traffic dynamics/routing impacts network structure
  - Network structure impacts traffic dynamics/routing
- Robustness/fragility considerations only make sense in the context of the broader system, i.e., protocol stack
  - Router-level: Inter-AS routing protocol
  - AS-level: Intra-AS routing protocol

New Mathematical Challenges

- Dynamics of and over networks
- Robustness/fragility of networks
- Multiscale network representations
- Networks of networks

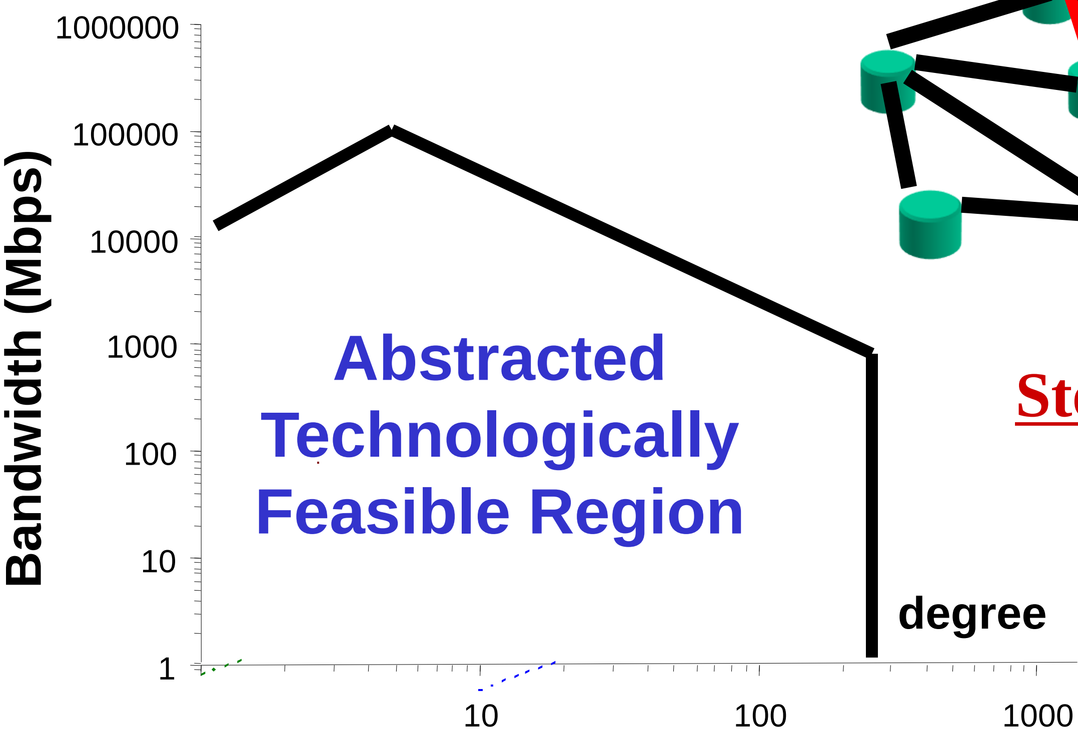
Dynamics of/over Networks

- PA-type modeling perspective
  - Networks grow by addition of new nodes/links according to specific rules
  - Limit networks as models of large-scale real-world graphs
  - Dynamics of networks without dynamics over networks
- Current mathematical efforts
  - Limits of graphs (Chayes et al. 2005)
  - Convergent graph sequences (Chayes et al. 2007)

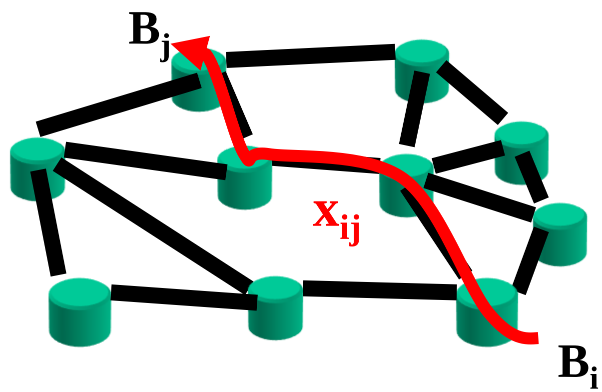
Heuristically Optimized Topologies (HOT)

Given realistic technology constraints on routers, how well is the network able to carry traffic?

Step 1: Constrain to be feasible



Step 2: pick traffic demand model



$$x_{ij} \propto B_i B_j$$

Step 3: Compute max flow

$$\max_{\alpha} \sum_{i,j} x_{ij} = \max \sum_{i,j} \alpha B_i B_j$$

$$s.t. \sum_{i,j:k \in r_{ij}} x_{ij} \le B_k, \forall k$$

Dynamics of/over Networks (cont.)

- HOT modeling perspective
  - Networks evolve in response to changing conditions
  - Traffic demands, technology, economics, government regulations, …
  - Evolution of node-internal structure
- Important observation
  - Coupling of network traffic and network structure
  - Network traffic/routing impacts network structure
  - Network structure impacts traffic flow/routing

Dynamics of/over Networks (cont.)

- HOT view leads to natural separation of timescales
  - Fast (traffic demands): traffic engineering (e.g., re-computing link weights)
  - Medium (traffic trends): network provisioning (e.g., adding links)
  - Slow (technological changes): network design (e.g., re-optimize)
- New mathematical approaches
  - Network utility maximization (NUM) (M. Chiang, S. Low, J. Doyle)

Network Robustness/Fragility

- On the one hand, the Internet is extremely robust to the loss of links/nodes
  - By design: #1 requirement of the original architectural design of the Internet
  - Via specific protocol (IP) that "sees failures and routes around them"
- On the other hand, the Internet is highly fragile to hijacking/attacking the very infrastructure that provides the existing robustness to the higher layers
  - Has little to do with connectivity, but is all about protocols
  - By design: Designers of the original Internet assumed "trust anybody"
  - This trust model is completely broken (spam, worms, viruses, denial-of-service attacks, botnets, etc.)

Network Robustness/Fragility (cont.)

- Network robustness is more than knocking out nodes/links …
- A relevant mathematical treatment of the Internet's robustness/fragility properties only makes sense in the context of the entire TCP/IP protocol stack …
- …, but the incorporation of the associated relevant mechanisms poses significant challenges for any (semi-) rigorous mathematical study of the robustness/fragility of today's Internet.

Network Robustness/Fragility (cont.)

- Key characteristics of large-scale, highly engineered systems
  - Highly structured, elaborate internal configurations
  - Layers of feedback and signaling
  - Robust to uncertainties in their environment/components
  - Vulnerable to rare or unanticipated perturbations
- The “robust yet fragile” nature of the Internet
  - Inevitable result of fundamental tradeoffs
  - Spiral of increasing complexity
    - To suppress unwanted/newly found vulnerabilities
    - Take advantage for increased performance
    - Added complexity leads to new vulnerabilities
- In desperate need for a theory that can provide guidance
  - Initial attempts: J. Doyle, P. Parrilo

Multiscale Network Representations

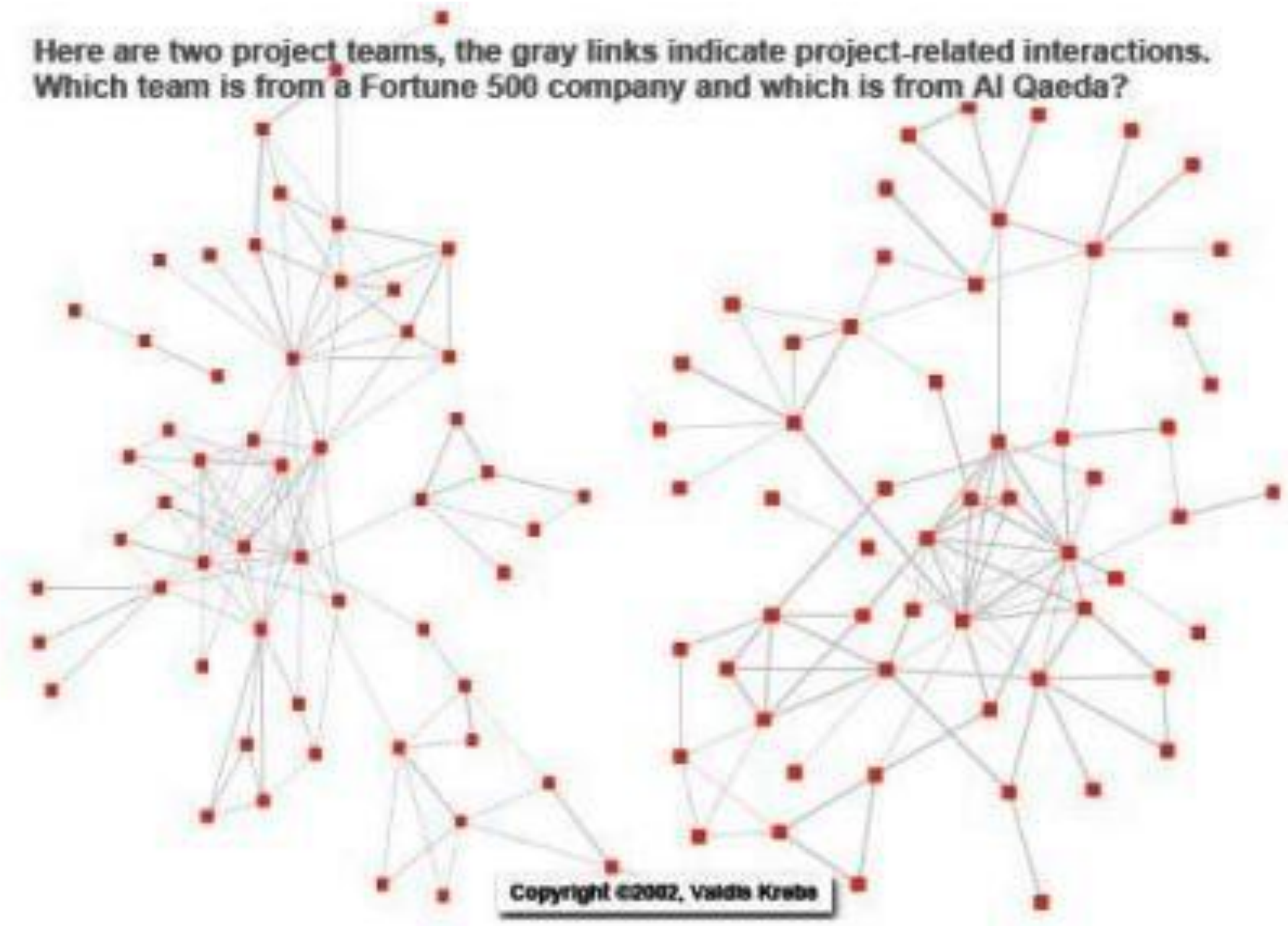
- Some natural hierarchical representations
  - Router-level (i.e., physical infrastructure)
  - PoP-level (Point-of-Presence, router clusters)
  - ISP-level (i.e., Internet Service Providers)
  - AS-level (i.e., Autonomous Systems)
- Associated traffic demands (traffic matrix)
  - Recent success: Router-, PoP-level
  - Unknown: ISP-, AS-level
- Any multi-scale treatment of the Internet must respect these naturally occurring hierarchical structures in today's Internet.
- New mathematical approaches
  - Diffusion wavelets (M. Maggioni, R. Coifman, P. Jones)

Networks of Networks

- Most of the infrastructure systems we rely on in our daily lives are designed and built as networks
  - Today's Internet (some 30,000 Autonomous Systems)
- Other examples of critical infrastructures
  - Electrical Power grid
  - PTN, TV/Radio, CATV, Wireless
  - Banking, finance, airline transportation, government
- New Vulnerabilities
  - Interconnected, interdependent
  - Internet: "central nervous system"
  - Catastrophic, cascading failure events; deliberate attacks
- The general allure
  - "Typical" behavior is often simple, suggesting naïve models
  - "Atypical" events reveal the role of enormous internal complexity

Further Implications of this Engineering Perspective

- The importance of “structure” vs. “function”


Here are two project teams, the gray links indicate project-related interactions.
Which team is from a Fortune 500 company and which is from Al Qaeda?
Copyright ©2002, Valdis Krebs

Further Implications of this Engineering Perspective

- The importance of “structure” vs. “function”

- Key question #1: What is the network as whole trying to achieve?
  - Internet router-level: see earlier
  - Internet AS-level: ?
  - WWW, P2P: ??
  - Social Networks: ???

- Key question #2: How is the network trying to achieve its objective?
  - Centralized
  - Decentralized, distributed (duality gap?)

On Modeling the Internet AS-level connectivity

- Applying our litmus test for network models
- Traditional AS connectivity modeling
- Criticism of traditional AS connectivity modeling
- An alternative approach based on engineering considerations

Conventional Approaches to AS-level Topology Modeling

- Step 1: Take the available measurements at face value
- Step 2: Analyze the data as if they could provide the ground truth about the Internet's actual AS-level connectivity structure
- Step 3: Propose a random graph model or construction that describes/fits the inferred AS maps well
- Step 4: Argue for the validity of the proposed model on the basis that it is capable of reproducing certain empirically observed properties of the inferred AS maps

Criticism of Conventional Modeling Approach

- Measurements
  - Connectivity-related Internet measurements are of limited quality
  - BGP is not a mechanism by which ASes distribute connectivity information, but is a protocol by which ASes distribute the reachability of their networks via a set of routing paths that have been chosen by other ASes in accordance with their policies.
- Modeling
  - Inferred AS maps are in general dubious or useless, unless they are accompanied by strong robustness results that state whether or not the observed properties are insensitive to the known ambiguities inherent in the underlying measurements.
  - Chang et al. (2004)

An Engineering Approach to AS-level Topology Modeling

- Engineering perspective
  - Surely, deciding on whether or not to establish what type of peering relationship and with whom is not the outcome of a series of chance experiments conducted by the different ASes, but is largely based on economic arguments.
- Need to understand the critical roles played by
  - AS-specific traffic
  - AS-specific geography
  - AS-specific business model
- Recent alternatives to PA-type models
  - Chang et al. (2006)

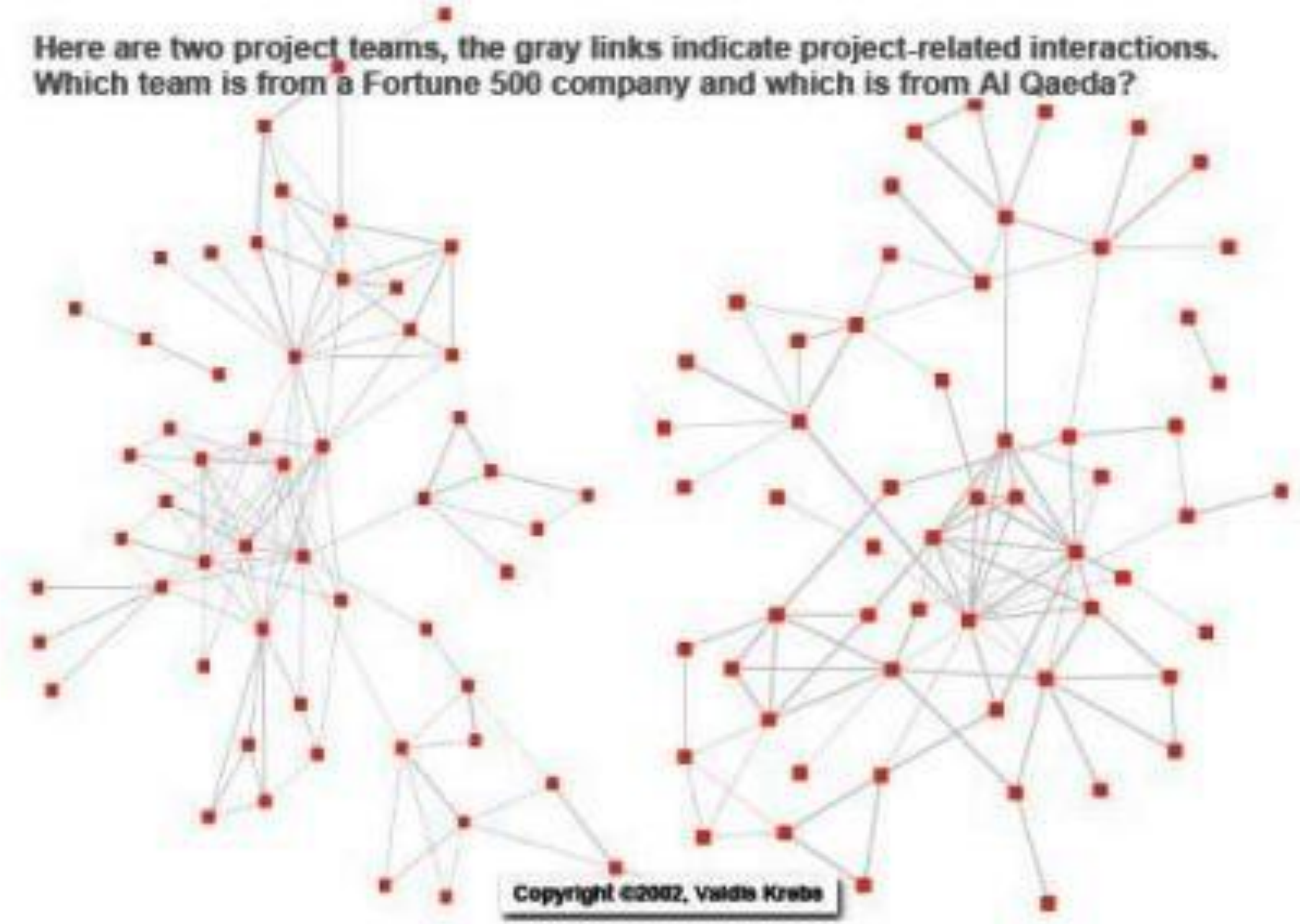
An Engineering Approach to AS-level Topology Modeling (cont.)

- Main challenges
  - What does the AS-level Internet as whole trying to achieve?
  - How does the AS-level Internet achieve its objective?
- Engineering approach
  - Optimization of tradeoffs between multiple functional objectives of networks
  - Subject to constraints on their components
  - With an explicit source of uncertainty against which solutions must be tolerant or robust
  - Node degree distribution is a non-issue
- What are the objectives, constraints, and main sources of uncertainty?

The Engineering Approach beyond Router- and AS-Connectivity

- Overlays
  - What does Gnutella/BitTorrent as a system trying to achieve? And how?
  - What is the Web as a whole trying to achieve? And how?
  - …
- What are the objectives, constraints, and main sources of uncertainty?
  - Technology, economics, socio-technological aspects, socio-economic factors, social science, user behavior, …
  - How to decide in a principled manner?

Functionality trumps Structure …



Main Problems with the "Network Science" Approach

- ✓ No critical assessment of available data
- ✓ Ignores all networking-related "details"
- ✓ Overarching desire to reproduce observed properties of the data even though the quality of the data is insufficient to say anything about those properties with sufficient confidence
- ✓ Reduces model validation to the ability to reproduce an observed statistics of the data (e.g., node degree distribution)

How to fix “Network Science”?

- ✓ Know your data!
  - ✓ Importance of data hygiene
- Know your statistics!
  - – Every dataset can be “mined” to yield power-laws
- ✓ Take model validation more serious!
  - ✓ Model validation $\neq$ data fitting
- ✓ Apply an engineering perspective to engineered systems!
  - ✓ Design principles vs. random coin tosses

The Main Take-Away Messages

The application of "Network Science" in its current form to the Internet has led to conclusions that are not controversial but simply wrong.

The application of "Network Science" to the Internet has become a textbook example for demonstrating what can and does go wrong if domain knowledge is ignored for the sake of hype and publicity.

There exists now an alternative approach to "Network Science" that provides a much-needed engineering perspective to balance the dominant statistical physics perspective in today's "Network Science".

And Don't Forget …

- Past: Modeling in the presence of high-quality data that can be taken at face value
  - *"All models are wrong … but some are useful"* (G.E.P. Box)

- Future: Modeling in the presence of highly ambiguous data that should not be taken at face value
  - *"When exactitude is elusive, it is better to be approximately right than certifiably wrong." (B.B. Mandelbrot)*
  - *In the case of the Internet, the scale-free network models are a textbook example of being "certifiably wrong", while the HOT models are an example of being "approximately right."*