

A strategic planning model for the operation plan in lamb supply chains

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Abstract: Operations research has been widely applied to support decision-making in the meat processing industry. Several tactical models have been proposed to develop plans for processing the carcasses into products. The common practice of a tactical model is that the domain experts provide multiple alternative cut patterns, and the model optimises the carcass allocation among the preset patterns. However, there remains a gap in identifying the profitable cuts from an extensive range of candidates under complex demands.

To address this challenge, we introduce the concept of a cut tree to present the cut logic involved in meat production. A cut tree is a Directed Acyclic Graph (DAG) that functions as a decision tree to reflect the rules governing the production of certain cuts that rule out others. By generating a set of cut logic constraints from the cut tree, we can represent all feasible combinations of products that can be made from a single carcass.

We then present an integer linear programming model to answer strategic questions about how to identify the most profitable cuts and optimise the allocation of carcasses to those cuts. We address a scenario in which the operator seeks to maximise the profitability of production subject to constraints on carcass availability, product orders, product size, and costs of trimming, while accounting for current wholesale prices.

The results of a case study demonstrate that our proposed model can increase operational profit in a boning room by at least 25% compared to randomly allocating carcasses. In addition, the model allows for greater flexibility in determining which products to make and how to allocate carcasses through a combination of constraints. As such, our proposed model has significant potential as a valuable research tool for scenario-based analysis, as well as for day-to-day operations in the industry.

Keywords: *Integer linear programming, cut logic constraint, lamb supply chain, meat production planning*

1 INTRODUCTION

The Australian lamb meat industry plays an important role in the country's food supply chain, serving both the domestic market and international exports. Lamb meat processing involves a complex set of operations that must be carefully coordinated in order to ensure that the final product meets customer specifications while maximising profits for the lamb processor. Within the lamb supply chain, the boning room plays a critical role in the production process, where the carcase is broken down into individual cuts of meat. One of the most challenging problems in the boning room is the planning and scheduling of operations to process the carcase efficiently and profitably. There are two levels of decision-making involved in this problem. The first is the strategic selection of the appropriate cut products to produce based on the availability of raw materials, seasonal demands, and customer requirements. The second level is the tactical decision-making which involves allocating the right carcasses to the appropriate product. Effective implementation of strategic and tactical decision-making processes in the boning room can help to enhance the efficiency and effectiveness of the lamb supply chain, and improve profitability.

Operations Research offers advanced analytical methods for decision-making, and has been successfully applied to a wide range of problems in the meat processing industry (Ahumada & Villalobos 2009, Plà-Aragonés 2015). In particular, optimisation models, including Linear Programming (LP), Mixed Integer Linear Programming (MILP), Stochastic Programming (SP), and Dynamic Programming (DP) have been used to support decision-making in meat processing, e.g., Rodríguez-Sánchez (2010), Soysal *et al.* (2014), Ge *et al.* (2022), Solano-Blanco *et al.* (2020), and Tabrizi *et al.* (2018). However, most of these works were proposed to improve tactical decision-making, and very limited literature exists to address strategic decision-making.

In this paper, we present a mathematical model for the optimal allocation of carcasses to lamb products in a lamb processing facility. Specifically, we developed an integer linear programming model to optimise the decision-making in both strategic and tactical levels simultaneously, which takes into account constraints on the physical relationship between lamb products, carcase availability, customer demands, market prices, and processing costs. This model covers a spectrum of situations faced by lamb processors. At one end of the spectrum, a specific quantity of each product can be specified (e.g., based on customer orders), and the optimiser has the flexibility only in deciding which carcasses to allocate for the given product. At the other end, the optimiser can have complete flexibility to determine both the product types and the carcase allocations, reflecting a situation where the operator can shift production towards a profitable market. Situations falling in between can be modelled by applying partial production constraints that reflect the degree of flexibility available to the operator. In all cases, the objective is to maximise profitability, which depends on input costs, predicted yields, and market prices. We conducted a case study consisting of five scenarios, and the results demonstrated that our proposed model can increase operational profit in a boning room by at least 25% compared to randomly allocating carcasses.

2 PROBLEM STATEMENT AND RELATED WORK

Lamb processing is composed of several stages, including pre-slaughter lairage, the slaughter floor, the boning room, packaging, storage, and dispatch. Each day, batches (or lots) of lambs arrive at the processing facility and are kept in pens. After the lambs are slaughtered, the carcasses, which are the animal bodies without the head, fore-limbs, blood and organs and with standard (legislated) levels of trim are kept in refrigerated rooms overnight before moving to the boning room. To simplify the optimisation process, carcasses are grouped into *carcasse types* based on weight and fat. In the boning room, the carcase is processed to obtain a range of meat cuts, which are defined as *cut types*, and the combination of cut types used to process a carcase is referred to as a *cutting pattern*.

Optimisation has been applied to a range of situations for the decision-making in meat processing. A MIP model was proposed in Jensen & Kjærsgaard 2010 for computing optimal sorting groups based on slaughter weights and fat layers of pig carcasses, including a new approach to handle measurement errors. However, this model was built on preset cutting patterns that did not address the decision-making at the strategic level. Furthermore, it only considered the production yield without including any of the costs, or customer demands. In Albornoz *et al.* 2015, a MILP model was developed to determine the production levels of pork products considering multi-products plans, multi-periods, batch quality distribution on carcasses and perishability. However, this model was also limited at the tactical level. It treated the operating cost as a pre-setting parameter associated with the cutting pattern and did not have the flexibility to calculate the costs for a arbitrary cutting pattern. In Kjærsgaard 2008, a MIP model was applied regarding the estimation of the economic consequences

of increased slaughter weight. The model consists of different alternative uses of pig carcasses at the strategic level, but did not provide the general form of the product processing constraints.

In Australia, the current operational practice is that the boning room manager develops the daily cutting plan manually based on the yield of products, carcass availability, the customer's demands, and operational constraints. While the boning room managers may rely on their expertise to allocate carcasses among different cutting patterns, there are numerous cut options available, each with varying yields and operating costs, and a complex cut logic exists that governs the production of certain cuts that rule out others. Thus, it's crucial to consider the cutting plan at both strategic and tactical levels that allocates the right carcass to the right cut. With complex customer demands, operational constraints, and rapid changes in the market, optimising the cutting plan becomes increasingly challenging.

3 MATHEMATICAL MODEL

The major question to be addressed herein is how to allocate the right carcasses from a given carcass population to the right cuts. The presented model aims to maximise profitability while considering constraints such as carcass availability and customer demands, including the desired cut quantity and size. The notations used in this model are described below:

$i \in [1, \dots, I]$	the i^{th} carcass type, where each index i maps to a (weight, fat score) pair
$j \in [1, \dots, J]$	the j^{th} cut types
N_i	the number of available carcasses of type i
n_j^c	the number of pieces of cut type j in one carcass
w_i^c	the carcass weight of carcass type i
f_i^c	the fat score of carcass type i
$w_{i,j}^{cut}$	the predicted weight of a piece of cut j that is got from a carcass of type i
$w_{i,j}^{fat}$	the predicted fat weight by-product to obtain one piece of cut j from a carcass of type i
$w_{i,j}^{trim}$	the predicted trim weight by-product to obtain one piece of cut j from a carcass of type i
$p_{i,j}^{cut}$	the wholesale market price (dollars per kilo) of cut type j , when produced from carcass type i
p^{fat}	the market price (dollars per kilo) of the fat by-product
p^{trim}	the market price (dollars per kilo) of the trim by-product
c_i^{grid}	the grid price (dollars per kilo) of carcass type i
c^{sla}	the slaughter charge per head
c^{pak}	the packaging cost per head
$t_{i,j}$	the time to procure a cut j from a carcass of type i
r^{bn}	the hourly rate of boning staff
$E(x)$	the revenue associate to a allocation x
$C(x)$	the cost associate to a allocation x
$x_{i,j}$	the (non negative integer) number of carcasses of type i to produce cut type j
x	an allocation vector containing all decision variables $x_{i,j}, i = 1, \dots, I, j = 1 \dots J$
C	the set of (constrained) feasible allocations

3.1 Cut tree

A *cut tree* is a novel concept introduced here. It is a useful tool to help translate the physical relationship between meat products and the processing procedure in the boning room to a set of mathematical constraints. A cut tree is a directed acyclic graph (DAG) that can be defined as a tuple (V, E, r) , where V is a set of vertices (nodes), E is a set of directed edges from the parent node to the child node, and r is the root vertex. The root node of a cut tree represents the *carcass*, the rest of the nodes represent cut types, and tracing the branches of the tree from left to right indicates which child cuts can be derived from a parent cut via further processing. There are two types of logic operating in the cut tree. The first logic type is AND, which indicates that at decision nodes, multiple cut types are procured simultaneously after one step of the process. A node with all edges connecting to child nodes being of the AND logic is called an AND node. An example of an AND node with two child nodes is shown in Fig 1a. Let's denote n as the number of products passed to the node for processing, x_1 as the number of cuts from parent node, and x_2 and x_3 as the numbers of cuts from child nodes after processing. The following relation holds for the AND node in Fig 1a:

$$x_2 = x_3, \quad x_1 + x_2 = n. \quad (1)$$

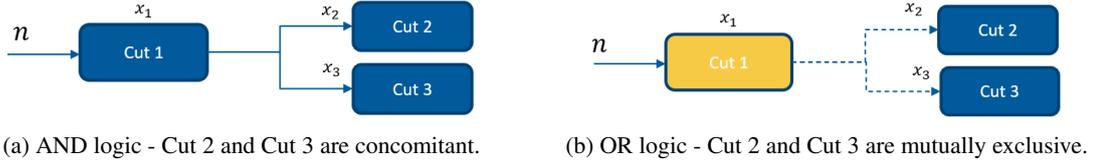


Figure 1. Two types of logic in the cut tree.

The second logic type is OR, which represents at the decision node only one cut type will be procured from a list of candidates. A node with all edges connecting to child nodes being of the OR logic is called an OR node. An example of an OR node with two child nodes is shown in Fig 1a, and the following equation holds:

$$x_1 + x_2 + x_3 = n. \quad (2)$$

In the real-world, there would be mixed logic in the meat processing. An example is shown in Fig 2a, one step further to process *Cut 1* will procure a piece of *Cut 2*, but only one piece of *Cut 3* or *Cut 4*. Here the node of *Cut 1* is neither an AND node or OR node. However, we can spilt the mixed logic and build a equivalent tree with only AND and OR nodes by adding dummy nodes as shown in Fig 2b.

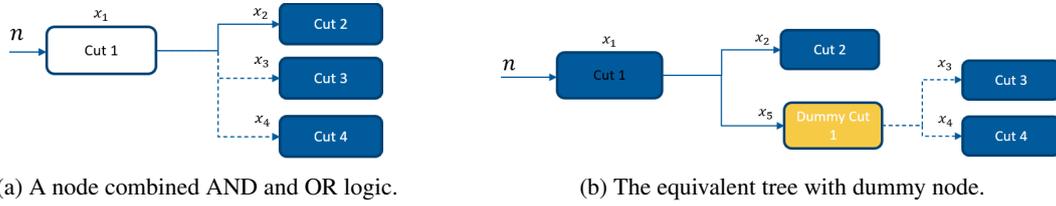


Figure 2. Mixed logic and the equivalent tree with the dummy node.

3.2 Cut logic constraints

From a cut tree that contains all cut products and the physical relationships that production of certain cuts require, we can generate the cut logic constraints which represents all feasible combinations of products that can be made from a single carcass. To illustrate the derivation of cut tree constraints, we consider the example of a cut tree depicting 9 possible cuts (for simplicity of exposition) shown in Fig 3. Let $n_i(x)$ be the number carcasses of type i that are processed as a result of the allocation x . For a given carcass type i , the following relations hold from equation (3a) to (3e).

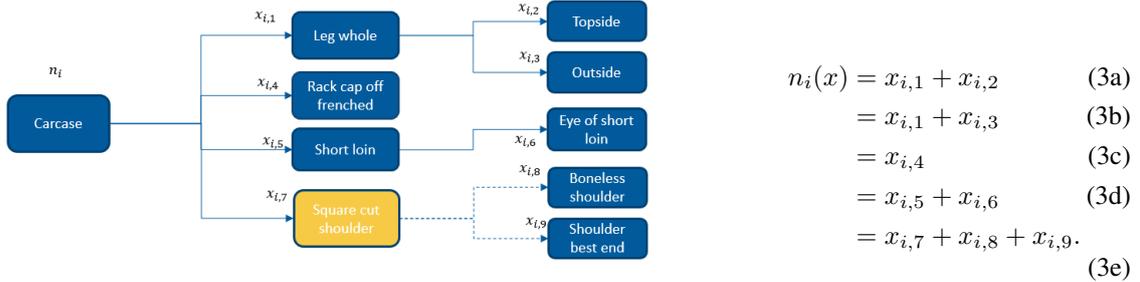


Figure 3. A example of cut tree.

These equations are based on the logic that the number of processed carcasses should be equal for all branches in a cut tree. Equations (3a) - (3d) reflect the AND logic along the top four branches of the tree, and equation (3e) is an example of the OR logic present in the lower two branches.

Define T_l to be the set of cut types that are obtained from processing a carcass through the l branch in the cut tree, and the collection of cut sets from all branches as $\mathbb{T} = \{T_1, T_2, \dots, T_L\}$. For the cut tree in Fig 3, we have $T_1 = \{1, 2\}$, $T_2 = \{1, 3\}$, $T_3 = \{4\}$, $T_4 = \{5, 6\}$, $T_5 = \{7, 8, 9\}$, and $\mathbb{T} = \{T_1, T_2, T_3, T_4, T_5\}$. We arbitrarily chose a cut set T_a as the ‘anchor’ branch, and let $\mathbb{T}^* = \mathbb{T} \setminus T_a$ denote the collection of cuts excluding the anchor branch. We can then write the equality constraints in the general form:

$$\sum_{j \in T_a} x_{i,j} = \sum_{j \in T_l} x_{i,j}, \quad \text{for } T_l \in \mathbb{T}^*, \quad i = 1, \dots, I. \quad (4)$$

Furthermore, the number of carcasses must not exceed the number available, hence:

$$n_i(x) = \sum_{j \in T_a} x_{i,j} \leq N_i, \quad \text{for } i = 1, \dots, I. \quad (5)$$

Let \mathcal{C}_{tree} denote the set of allocations x which satisfy the cut logic constraints (4) and (5).

3.3 Revenue

Revenues are considered from three sources. The first source is the income from selling meat products, which is the dominant part of the revenue model. The remaining two are fat and trim produced during the cutting process. These products are less valuable compared to the meat products, but still bring income when they are sold on the market. Given an allocation x , the revenue from the meat processing is:

$$E(x) = \sum_i \sum_j (p_{i,j}^{cut} w_{i,j}^{cut} + p_{i,j}^{fat} w_{i,j}^{fat} + p_{i,j}^{trim} w_{i,j}^{trim}) n_j^c x_{i,j}. \quad (6)$$

where n_j^c is the number of pieces of a cut obtained from a carcase (e.g., we will get one piece of neck and two pieces of leg products from a single carcase.)

3.4 Cost

The procurement costs consist of four parts; the purchase cost, slaughter cost, packaging cost and boning cost. Here, we consider a flat rate slaughter charge for each carcase. Similarly, we assume a per carcase packaging cost is available as a fixed initial estimate, and spread this cost to each product in proportion to its predicted cut weight. Furthermore, the time to produce each cut type were estimated by the domain expert. Adding the cost to purchase live lamb, the overall cost given a allocation x is:

$$C(x) = \sum_i \sum_{j \in T_a} w_i^c p_i^{grid} x_{i,j} + \sum_i \sum_{j \in T_a} c^{sla} x_{i,j} + \sum_i \sum_j c^{pak} \frac{w_{i,j}^{cut}}{w_i^c} n_j^c x_{i,j} + \sum_i \sum_j r^{bn} t_{i,j} n_j^c x_{i,j}. \quad (7)$$

Note that the first term in (7) is the purchase cost, the second term is the slaughter cost, the third term is the packaging cost and the last term is the boning cost.

3.5 Product constraints

To meet customer demands and operational limits, different types of restrictions may be imposed on the products, such as constraints on the quantity of cuts, weight of cuts, and weight of carcasses.

Let n_j^{min} and n_j^{max} be the minimum and maximum number of pieces of cut type j to produce respectively. We impose the cut quantity constraints:

$$n_j^{min} \leq \sum_i x_{i,j} n_j^c \leq n_j^{max} \quad \text{for } j = 1, \dots, J. \quad (8)$$

A fixed order quantity Q_j can be specified for a given cut type j by setting $n_j^{min} = n_j^{max} = Q_j$. Let $\mathcal{C}_{cutquantity}$ denote the set of allocations x which satisfy the constraints (8).

To accommodate packaging size requirements, optional restrictions can be set on the weight of produced cuts. For a given cut type j , let's restrict the product cut weights to lie between w_j^{min} and w_j^{max} kg. Letting

$$\mathcal{S}_j = \{i \mid w_{i,j}^{cut} < w_j^{min} \vee w_{i,j}^{cut} > w_j^{max}\}, \quad (9)$$

we add the cut weight constraints:

$$x_{i,j} = 0 \quad \text{for all } i \in \mathcal{S}_j. \quad (10)$$

Let $\mathcal{C}_{cutweight}$ denote the set of allocations x which satisfy the constraints (10) for all j as required.

We can also set optional restrictions on the carcase weights that are allowed to be used to make a particular cut type j . This restriction is intended to be used for modelling existing carcase weights based 'rule of thumb' allocations. Suppose that the cut type j must only be produced from carcasses with weights that lie between r^{min} and r^{max} kilograms. Letting

$$\mathcal{U}_j = \{i \mid w_i^c < r^{min} \vee w_i^c > r^{max}\}, \quad (11)$$

we add the carcase weight constraints:

$$x_{i,j} = 0 \quad \text{for all } i \in \mathcal{U}_j. \quad (12)$$

Let $\mathcal{C}_{carweight}$ denote the set of allocations x which satisfy the constraints (12) for all j as required.

3.6 Statement of optimisation problem

Our optimisation problem can be stated as

$$\begin{aligned} & \max_x \quad E(x) - C(x) \\ & \text{subject to} \quad x \in \mathcal{C}_{tree} \cap \mathcal{C}_{cutquantity} \cap \mathcal{C}_{cutweight} \cap \mathcal{C}_{carweight}. \end{aligned} \tag{13}$$

Note that both the objective function and the constraints are linear, therefore the optimisation model is an integer linear programming problem.

4 CASE STUDIES

In this section, a case study is presented to illustrate the advantage and flexibility of the proposed model. A realistic cut tree consisting 122 cuts was applied. It was built by the domain expert and represented the real-world cut logic applied in the industry. The carcase data of 1000 carcasses were obtained from a collaborating abattoir. Carcasses were categorised into 265 carcase types (bins), with the combination of carcase weight (13kg to 39kg with a 0.5kg step) and fat depth (3mm to 23mm with a 5mm step).

Cut weight, fat weight and trim weights for each selected cut were generated from the lamb value calculator of Hocking-Edwards *et al.* (2015). Wholesale prices of cuts, trim and fat, carcase grid prices, slaughter charges, packaging costs, boning labour rates, and estimates of cuts boning time were obtained from industry sources. In this study, we set up three different cut options in the forequarter, loin, and hindquarter regions, and two different cut options in the rack and flap regions which were representative of lambs processed for domestic market in Australia (Tab 2).

Table 2. The setting of cuts options, cuts quantity, weight range and carcase weight range in the case study (Note different regions of the same carcase can be allocated to different cut options).

Region	Option 1				Option 2				Option 3			
	Cuts	Min cut pieces	Cut weight (kg)	Carcase weight (kg)	Cuts	Min cut pieces	Cut weight (kg)	Carcase weight (kg)	Cuts	Min cut pieces	Cut weight (kg)	Carcase weight (kg)
FQ	Square cut shldr	400		<22	Best end shldr	400	1~2.2	22~26	Boneless shldr	400	>1.5	>26
Loin	Short loin eye	400	>0.4	>28	Short loin (no tail)	400	>0.6	<27.7	Short loin 50mm tail	400	<1.4	<27.7
Rack	French rack	200	>0.6	>26	Rack	600		≤26				
Flap	Flap	500	>1.2	>26	Abdominal flap	500		≤26				
HQ	Leg chump on	400	<2.75	<22	Round	300	>0.55	>26	Leg chump off	400	>2.5	22~26

In the first scenario, we did not apply any product constraints, and let the optimiser select the best plan satisfying the cut logic. In real-world operations, the boning room manager has a list of demands from customers either on product quantity, or on product size. In the second scenario, we ran the model by adding the cut quantity constraints on some of the cuts as shown in the column of ‘Min cut pieces’ in Fig 2. On the other hand, in order to restrict the product size, the boning room managers usually group the carcasses by carcase weight and assign a cutting pattern for each group based on the prior experience. In the third scenario, we simulated the boning room manager’s strategy, and added the carcase weight constraints on some of the cut options as shown in the column of ‘Carcase weight (kg)’ in Fig 2. Instead of restricting the cut size by limiting carcase weight, the proposed model can directly apply the cut weight constraints (10) and let the optimiser discover the best plan. In scenario four, we ran the optimisation by adding cut weight constraints on some of the cut options as shown in the column of ‘Cut weight (kg)’ in Fig 2. The proposed model has the flexibility to combine multiple types of constraints, and we combined the cut quantity constraints and cut weight constraints and ran the experiment in scenario five. By contrast, we randomly allocated carcasses to the available cut options for the five scenarios. We ran the random allocation 100 times for each scenario to get the mean value of the profits from random allocation to compare with the optimised profits.

The resulting models of this study include 32330 variables and up to 65086 constraints. The modelling software was *Google OR-Tools* with python wrapper. All instances of the study were solved by a PC with i5-6300U CPU @ 2.40GHz and 16G RAM and the average runtime was 0.72 seconds.

Table 3 shows the profit results obtained from the proposed model and random allocations. The results clearly indicated that the profit generated from our model outperformed the random allocation in all scenarios. The

Table 3. Comparison of the allocation profits.

Scenario	Optimised profit	Random allocated profit
1.No product constraints	\$46,018.70	\$15,253.43
2.Cut quantity constraints	\$32,214.91	\$15,855.86
3.Carcase weight constraints	\$22,167.77	\$17,723.13
4.Cut weight constraints	\$29,202.94	\$4,407.85
5.Cut quantity & cut weight constraints	\$21,475.31	\$7,313.88

optimised plan in scenario one yielded the highest profit as the optimiser had more flexibility to adjust the production towards profitable cuts. The profit decreased when cut quantity, cut weight and carcase weight constraints were added to the model as the solution space was reduced. The most realistic case in the experiments is the random allocated plan with the carcase weight constraints, which closely resembled the strategy used by boning room managers in abattoirs. By applying our model with the same carcase weight constraints, the optimised plan generated a 25% increase in profit compared to random allocation. Moreover, the profit further increased when cut weight constraints were directly applied.

5 CONCLUSION

In this paper, we propose an integer linear programming model to address the strategic question of how to optimise cutting plans by allocating the appropriate carcasses to the right cuts. We introduce the concept of a cut tree to illustrate the cut logic of meat production and generate a set of cut logic constraints that reflect the feasible combinations of products that can be made from a single carcase. We then consider a scenario where the operator aims to maximise profitability subject to constraints on carcase availability, product orders, product size, trimming costs, and wholesale prices. The optimiser can serve as a research tool for scenario-based analysis, such as predicting the allocation changes required to optimise profits in response to changes in market prices, or it can be utilized in day-to-day operations to support the strategic decision-making of lamb processors. We have developed a web-based application based on this model that can run the optimisation model and compare the results for different inputs and scenarios. Further research could focus on developing algorithms to automatically generate cut logic constraints from a cut tree or simplify the cut tree with duplicate cuts in different branches.

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