Variational Methods and Optimal Control
Class Exercise 5: due before lecture, on Thursday 18th October, 2012

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1. **Transversals:** Find the coordinates of the point(s) nearest the origin on the surface $xyz = a^3$, for $x, y, z \geq 0$.

   Show (using the transversal conditions and the Euler-Lagrange equations) that if we were to draw a line between this point and the origin, it would be a transversal of minimum length between the origin and the surface.

2. **Optimal Control:** Minimize

$$F\{u\} = \int_0^1 u^2 \, dt$$

   subject to

$$\begin{aligned} \dot{x_1} &= u - x_2 \\ \dot{x_2} &= -u \end{aligned}$$

   and

$$\begin{aligned} x_1(0) &= 2 \\ x_1(1) &= 1 \\ x_2(0) &= 0 \\ x_2(1) &= 1 \end{aligned}$$

3. **Optimal Control:** Find the minimum value of

$$F\{u\} = x(1) + \int_0^1 \alpha u^2 \, dt,$$

   where $\alpha > 0$, $x(0) = 0$, $x(1)$ free, and

$$\dot{x} = u.$$

   How does the answer change if we add the condition that $|u(t)| \leq 1$?