Transform Methods & Signal Processing
Class Exercise 5:
hand in before lecture Friday 9th October

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Note, questions marked by a (*) are harder than normal questions, and are for masters students. Bonus marks may be awarded to other students who solve these.

1. $\boxed{\text{10 marks}}$ Figure 1 shows a box diagram of a biquad filter.

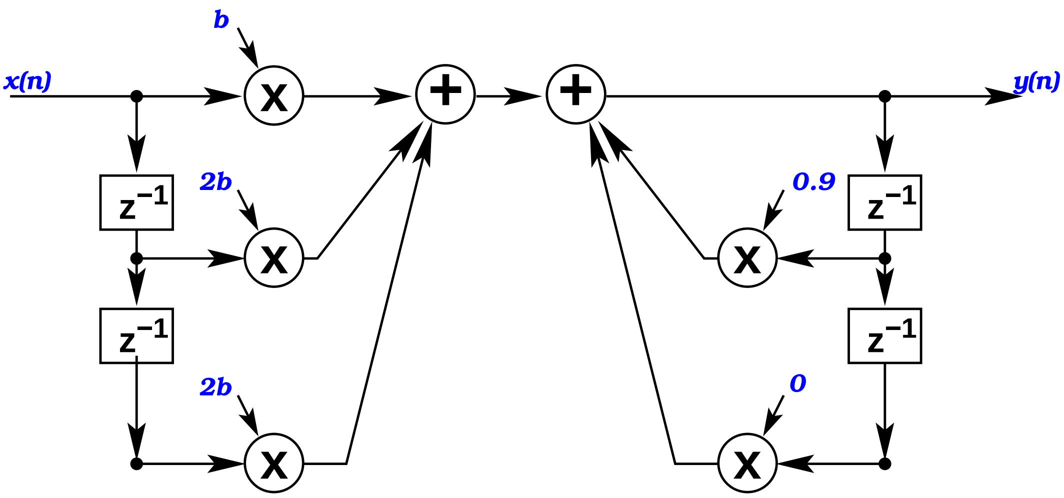

Figure 1: A biquad filter.

(a) Write the z-transform form of the transfer function of the filter.
(b) Find the poles and zeros of the filter.
(c) Is the filter stable? Why?
(d) Is the filter invertible? Why?
(e) Is the filter a high-pass, band-pass, or low-pass filter? Why?
(f) Find (analytically) the impulse response of the filter.
(g) Is the filter linear? time-invariant? causal?
(h) What would be the result of passing a signal through two such filters? Draw a box diagram of the new filter (in the same form as that of Figure 1, i.e. not as a cascade of two biquads).

2*. $\boxed{\text{10 marks}}$ A filter is BIBO (Bounded Input, Bounded Output) stable iff the impulse response is absolutely summable, i.e., its L^1 norm exists and is finite, e.g.

$$\sum_{i=-\infty}^{\infty} |w(i)| < \infty.$$

(a) Prove that this is a necessary condition for a filter with $w(i) \geq 0$ to have bounded output for all bounded input signals. Explain how we can generalize to the case where some $w(i) < 0$.

(b) Prove (using the triangle inequality) that this is a sufficient condition for BIBO.

(c) It is often convenient to characterize BIBO filters in the frequency domain (i.e. using a z-transform), i.e., by noting that all of the filter's poles must be inside the unit circle (in the complex plane). The proof revolves around showing that the z-transform of a BIBO filter must converge on the unit circle – show this is the case.