ADELAIDE UNIVERSITY
AUSTRALIA

Examination in School of Mathematical Sciences

Semester 2, 2004

4485	Modelling Telecommunications Traffic
	APP MATH 4014

Official Reading Time: 10 mins
Writing Time: 180 mins
Total Duration: 190 mins

NUMBER OF QUESTIONS: 5 TOTAL MARKS: 100

Instructions

- Answer ALL questions.
- Begin each answer on a new page.
- Examination materials must not be removed from the examination room.

Materials

- 1 Blue books are provided.
- Calculators are permitted.
- Closed book examination.

DO NOT COMMENCE WRITING UNTIL INSTRUCTED TO DO SO

1. Consider a Continuous-time Markov Chain (CTMC) with the following transition matrix

$$Q = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

   (a) Write down and solve the equilibrium equations for this CTMC.

   (b) Write down the probability transition matrix P^j corresponding to the jump chain of the above CTMC.

   (c) Find the equilibrium distribution of P^j.

   (d) Do the two equilibrium distributions calculated differ? Comment on why this is the case. Is this unusual?

[15 marks]

2. Consider an arrival process that can be in three separate phases. In each phase single arrivals occur after an exponentially distributed amount of time with rates 1.0, 2.0, and 3,0 respectively. No phase transitions occur when an arrival occurs. Transitions can occur between phase 1 and phase 2, with rate 10.0; between phase 2 and phase 3, with rate 5.0; and between phase 3 and phase 1 with rate 1.0. All of these transitions are exponentially distributed. This system is a Markov Process.

   (a) Write down the Q matrix that describes the arrival process. Do this in terms of the matrices that form Q.

   (b) What is the stationary distribution of being in the different phases of this Markov Process?

   (c) Determine the fundamental arrival rate of this process.

[10 marks]

3. Assume we are given the network shown in Figure 1, and told that that network uses shortest path routing, with the path length given by the number of hops. The 5 links in the network are bi-directional, and we label each direction as shown in Figure 1 with numerical labels from $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and the 12 possible end-to-end routes are listed in order as $(AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC)$.

   (a) A partial routing matrix A for the network in Figure 1, is given below. Complete the values for the row consisting of X's, and explain the meaning of this row.

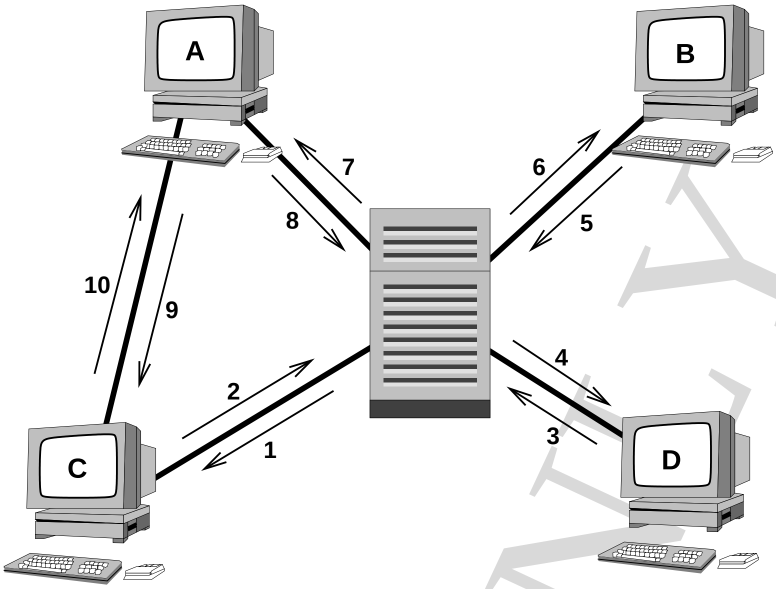

Figure 1: Simple traffic matrix problem. The numbered arrows denote the link labels.

$$
A = \begin{array}{c}
\begin{array}{cccccccccccc} AB & AC & AD & BA & BC & BD & CA & CB & CD & DA & DB & DC \end{array} \\
\left(\begin{array}{cccccccccccc}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
X & X & X & X & X & X & X & X & X & X & X & X \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}\right)
\end{array}
\begin{array}{l} \\ \text{link 1} \\ \text{link 2} \\ \text{link 3} \\ \text{link 4} \\ \text{link 5} \\ \text{link 6} \\ \text{link 7} \\ \text{link 8} \\ \text{link 9} \\ \text{link 10} \end{array}
$$

(b) In this network, imagine we have SNMP measurements of the traffic on each link, but we wish to estimate the traffic between end-points (assuming that traffic from a node to itself is zero). Express this problem as a linear inverse problem, and explain why the problem might be hard to solve.

(c) Given a traffic matrix describing the traffic between end-points defined by

$$
\mathbf{T} = \begin{pmatrix} 0 & T_{AB} & T_{AC} & T_{AD} \\ T_{BA} & 0 & T_{BC} & T_{BD} \\ T_{CA} & T_{CB} & 0 & T_{CD} \\ T_{DA} & T_{DB} & T_{DC} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 & 5 & 6 \\ 6 & 0 & 10 & 12 \\ 9 & 12 & 0 & 18 \\ 12 & 16 & 20 & 0 \end{pmatrix}
$$

write the traffic matrix as a column vector (using the same ordering as used in the routing matrix) and then determine how much traffic there would be on link 3.

(d) Determine from $\mathbf{T}$ the total volume of traffic into, and out-of the network at each end point T_A^{in} and T_A^{out} respectively, and then compute the traffic matrix we would estimate from a simple gravity model.

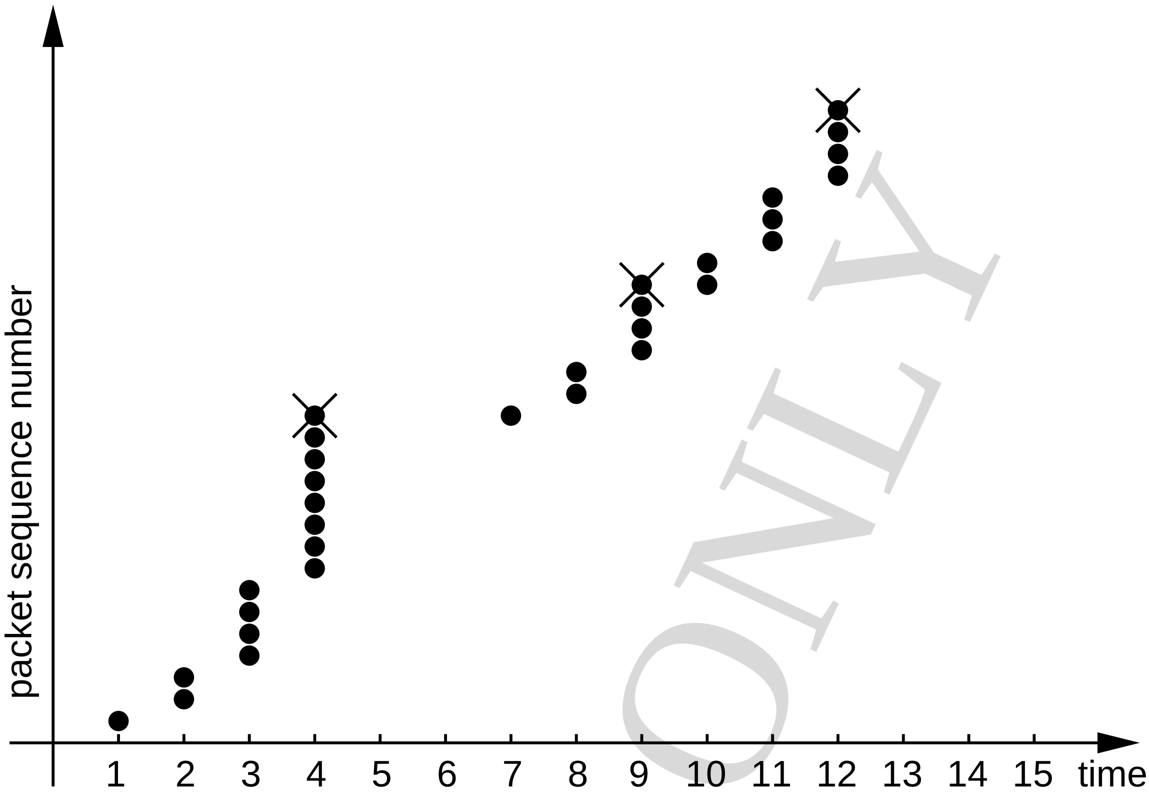

Figure 2: TCP congestion control example.

(e) We note that the results of the gravity model aren't the same as the actual traffic matrix. Explain (in a few sentences) a general approach to including the information from the link measurements to refine the accuracy of this estimate. [Note, you do not need to provide detailed mathematics of the approach used, only an intuitive description of this approach].

[20 marks]

4. (a) Figure 2 shows a stream of packets from a single TCP source. The graph shows the packet sequence number vs the time at which the packet is sent. Crosses indicate packets that are dropped in transit.

   1. Draw graphs showing the size of the congestion window (in packets) at each time interval, and similarly the size of the slow-start threshold, assuming that this starts at ∞.
   2. Assuming TCP Reno is used, describe what is happening at each time point, and why it happens.

   (b) Assume that packet losses occur randomly, with each packet being lost with probability p, independent of the other packets, and the window size. Model the window size in the congestion avoidance phase by a series of probabilistic recurrence equations, and from this, determine a simple relationship between the loss probality and the window size during congestion avoidance (for greedy traffic sources, and small loss probability p). Assuming a round-trip time of R, and an average segment size of M, determine the average throughput of such a connection.

(c) What effect might delayed acknowledgements have on the above results.

(d) Assume that the average throughput on a link is given by the law

$$x = \frac{M}{R\sqrt{p}}$$

where M is the average size of packet, and R is the round-trip time. Assume that he loss rate on the link is given by the approximation

$$p = \frac{[x-c]^+}{x}$$

where $[\cdot]^+$ denotes the positive part. Now determine the packet loss probability one might expect in equilibrium by using a fixed-point equation.

5. Write an essay on **one** of the following two topics:

   (a) Describe the key principles underlying TCP's operation, in particular, describe briefly the goal, and approach of the congestion control, but also note the end-to-end principle, and robustness principle, and describe.

   (b) What are do you need to get right in modelling Internet traffic? In particular, how does measurement effect modelling? What principles need to be applied when building models? Is a model automatically better if it fits a dataset better? What features are commonly observed in Internet traffic and why are they important?

   Your essay should be no longer than one page, and clear, concise presentation will be taken into account in marking. [20 marks]