Practical 6: Submit solutions in Matlab Grader before Fri 1st November at 5pm.

Bring your lecture notes with you for reference.

The aim of this practical is to try out Branch and Bound. We will proceed using MATLAB's `linprog` to solve the sub-problems, to avoid all of the minutiae of each relaxation.

Tasks Consider the Integer Linear Program (ILP)

$$\begin{array}{rrcrcrcl} \max z = & 2x_1 & + & 10x_2 & + & 1x_3 & & \\ \text{s.t.} & & & & & & & \\ & 6x_1 & + & 4x_2 & + & 4x_3 & \leq & 12 \\ & 4x_1 & + & 7x_2 & + & 3x_3 & \leq & 18 \\ & & & x_2 & + & 4x_3 & \leq & 17 \end{array}$$

with $x_i \geq 0$ and integer for all i.

1. Input the matrices and vectors A, $\mathbf{b}$ and $\mathbf{c}$ into MATLAB, and use `linprog` to solve the relaxed LP corresponding to the ILP above. **Check your solution in Matlab Grader.**

   Hints:

   (a) Write your solutions into a MATLAB `.m` script, so that you can add to it as you need for the following questions, and so that you can just cut and paste your solutions into Matlab Grader.

   (b) The Grader template will give you some ideas about how to format or structure your results.

2. Based on your solution to the relaxed problem branch on the variable x_2 by creating two new IPs with extra constraints

   $$x_2 \leq \lfloor c \rfloor \quad (1)$$
   $$x_2 \geq \lceil c \rceil \quad (2)$$

   where c is the value of x_2^* from the previous relaxation.

   Hints:

   (a) You don't have to change $\mathbf{c}$ at all, and you don't have to completely retype your constraint matrix A or the vector $\mathbf{b}$.

   (b) We could have replaced the above inequalities with equalities, but don't for comparison to Matlab Grader.

   Check your new matrices, A_i, $\mathbf{b}_i$ and $\mathbf{c}_i$, where $i = 1$ or 2 depending on which constraint you add from the above two, by entering them in Matlab Grader.

3. Solve the two new problems, again using `linprog`.

   Check your solutions in Matlab Grader.

   Hints:

   (a) Make sure to get all the output from `linprog`, so you can test if the problems are feasible as well as what the solution is.

4. Follow the Branch and Bound Rules to analyse the results, and make appropriate further branches or fathom the branch, following down the Branch and Bound tree until you can determine the solution to the ILP.

   Check your final result in Matlab Grader.