Assignment 4: Due Monday Sept 16th at 3pm.

Assignments to be handed in through MyUni. Please ensure written assignments are clearly legible. Typed assignments are preferred. Some help may be given in practicals to help get you started with Overleaf/LaTeX in order to present your work well.

1. Consider the LP given in our first problem. Consider what would happen if we made a poor prediction of the profit obtained for a bed.
   (a) Plot charts of your solutions $\mathbf{x}$ and profit z as a function of the correct profit (on a bed).
   (b) How large and small a profit (on a bed) could we make before the optimum number of each item changes?
   (c) Comment on the sensitivity of this problem to such errors.

   [Hint: you will want to use Matlab to find the solutions, as you will need to solve for a large number of LPs]

2. Imagine a sports competition, which starts with a round-robin phase where every team plays every other team once.

   There are 5 teams, and 2 playing grounds. Determine an Integer Program to find a schedule of games with the minimum number of possible rounds.

   Hints: label the teams $\{1, 2, \ldots, n\}$, and then
   (a) Use the variables x_{ijk} where

$$x_{ijk} = \begin{cases} 1, & \text{if team } i \text{ plays } j \text{ in round } k \\ 0, & \text{otherwise} \end{cases}$$

   How many variables do we need?
   (b) Write mathematical constraints for the problem, *i.e.,* translate the following into mathematics.
   - Each team must play each other team exactly once.
   - There are only two grounds, so only two games can be played in each round.
   - No team can play more than once in a particular round.

   (c) We will minimise the number of rounds by taking a value of K larger than we need, and penalising the later games, *i.e.,* we will minimise some weighted function of the variables, such that later games cost more. Write this mathematically.

   N.B. you do not have to solve the problem (yet).

   Hint: we often use *summation notation* when specifying an *abstract* optimisation problem, *i.e.,* a problem specified in general terms rather than one where each variable refers to a specific "item". For instance, in the above, we talk about 5 teams, but it is more powerful to write your solution in terms of n teams, using general notation. We could write constraints, such as, all teams must play 3 games as

$$\sum_{j=1, j\neq i}^{n} \sum_{k=1}^{K} x_{ijk} \geq 3, \text{ for all } i,$$

   which says that for all teams i, the sum of games i plays with the other teams j and in rounds k must be greater than or equal to 3.

3. Derive in Big-O notation, the computational cost of the following algorithm, given some input vector $\mathbf{x}$ of length n.

```
total = 1 + 3;
while n > 0
  if x(n) > 0
     total = total + x(n)
     n = n-1;
  else
     total = total - x(n)
     n = n-2;
  end
end
```

   Hints
   (a) Verify that the loop always terminates.
   (b) What is the worst case?
   (c) In the worst case, how many times do we invoke the loop, and how many operations occur each time?
   (d) What is the total number of operations?
   (e) Convert your result to Big-O notation.

4. You are told that Algorithm A takes $O(n)$ time, and Algorithm B takes $O\big((\log n)^6\big)$ time.
   (a) Given everything else is equal, which algorithm seems preferable and why?
   (b) List three reasons why this assessment might be naïve.

[20 marks]