Assignment 6: Solutions

TOTAL MARKS: 10

1. - No solution attached, but note that the files included locations for each node (each was an instance of a Waxman random graph). Many GraphML readers can't cope with extra data such as this (e.g., Julia can't at present). Waxman parameters for each dataset are given below.
   - Solutions:
     (a) a1898629: $s = 7.24379$, $q = 0.17227$
     (b) a1897413: $s = 4.85705$, $q = 0.10274$
     (c) a1932791: $s = 4.92125$, $q = 0.10684$
     (d) a1871789: $s = 6.74700$, $q = 0.18893$
     (e) a1734046: $s = 2.88459$, $q = 0.10498$
     (f) a1820798: $s = 6.10699$, $q = 0.18635$
     (g) a1825938: $s = 7.70884$, $q = 0.18748$
     (h) a1813487: $s = 4.67488$, $q = 0.19184$
     (i) a1871781: $s = 6.02077$, $q = 0.14768$
     (j) a1907611: $s = 2.11689$, $q = 0.18222$
     (k) a1709218: $s = 2.03428$, $q = 0.19523$
   - Marked according to content and presentation.

[8 marks]

2. A distance metric must satisfy the following properties:

1	non-negativity:	$d(x, y) \geq 0$
2	identity:	$d(x, y) = 0 \Leftrightarrow x = y$
3	symmetry:	$d(x, y) = d(y, x)$
4	triangle inequality:	$d(x, z) \leq d(x, y) + d(y, z)$

In both cases the distances are constructed by adding positive weights, and so properties 1 and 2 must hold. Note that identity is an equivalence: it is not enough to state that $d(x, x) = 0$, you also have to show that $d(x, y) = 0$ implies $x = y$ (which is true because any non-trivial path includes positive weights, but must be stated).

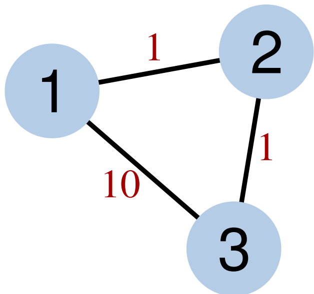
Property 3 holds because the graph is undirected, and hence path distances are symmetric. So the only difference between the two can lie in the triangle inequality.

(a) Presume that there are three nodes i, j and k such that

$$d(i, j) > d(i, k) + d(k, j),$$

there there is a shorter path from i to j than currently indicated by $d(i, j)$, and hence we have a contradiction. Thus the triangle inequality holds.

(b) We show the hop/weight combination to obtain distances cannot be a metric with a counter example (edge weights shown in red).



The minimum hop distance from node 1 to 3 is the direct edge between the two nodes. However, the distance of this is 10. Thus

$$d(1, 3) > d(1, 2) + d(2, 3),$$

and hence the triangle inequality is violated.

[2 marks]