Communications Network Design

lecture 09

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The Network Design Problem

In this lecture we consider a new optimization problem, the network design problem, where we can choose the network links (in contrast to routing where we only chose the routes across a given network). In this lecture we present some basics such as **star-like** topologies, **ring** topologies and the **travelling salesman's problem**.

In this lecture we consider a new optimization problem, the network design problem, where we can choose the network links (in contrast to routing where we only chose the routes across a given network).

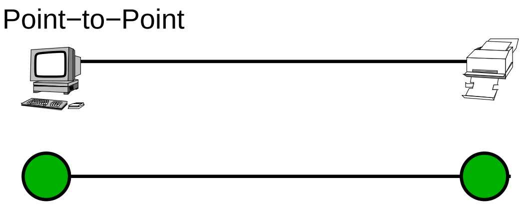
Network Design Problem

- the problems so far have concerned routing
  - network is given
  - we need to find optimal routing
- now we want to consider how to design the network
  - from scratch
  - routing is part of the design
- inputs
  - a set of nodes (locations)
  - forecasts of traffic demands

Example topologies

- point-to-point
- linear or bus
- ring
- hub and spoke or star
- double star
- fully connected (mesh) or complete topology or clique
- mesh
- (spanning) tree
- hybrid

Point-to-point



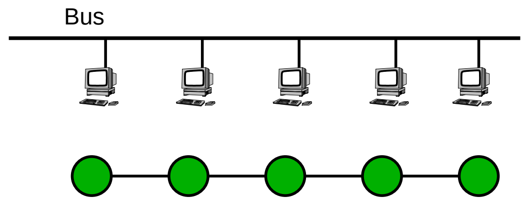
description: back-to-back connection of two nodes
examples:

- (old fashioned) printer connection
- serial link
- PPP (Point-to-Point Protocol)

comments:

- used as a component of a larger network

Bus



description: a single line (the bus) to which all nodes are connected, and the nodes connect only to this bus.
examples:

- physical structure of 10Base2 Ethernet
- logical structure of 10BaseT Ethernet with a hub

comments:

- design often matches a building (corridors)
- no redundancy (failures effect whole network)

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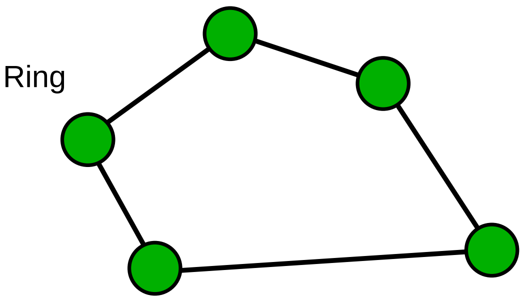
Ring



description: Every node has exactly two branches connected to it, so that they form a (logical) ring.
example:

- SONET, FDDI, Token Ring

comments:

- two paths provide some redundancy (a dual ring)

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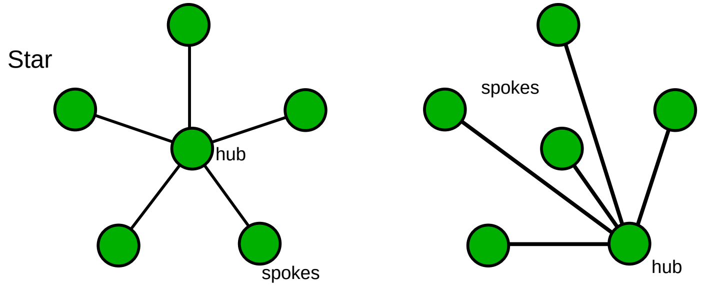
Star



description: peripheral (spoke) nodes are connected to a central (hub) node. All communications is via the hub.
examples:

- physical topology of 10BaseT Ethernet with a hub
- logical topology of 10BaseT Ethernet with a switch

comments:

- hub node failures are critical

Hub node failures are critical, but also note that any link failure will disconnect a node as well.

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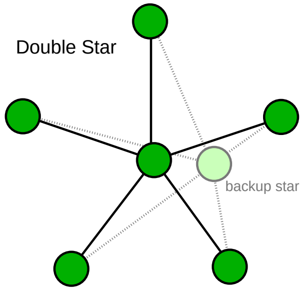
Double star



description: two stars, with two hubs, effectively, one is a redundant backup for failures.

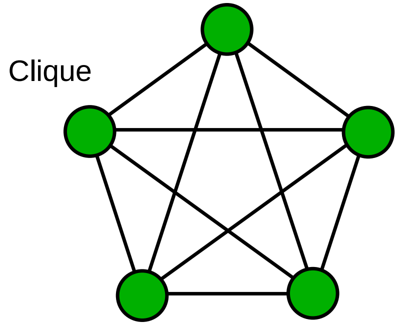
example:

- used for many networks

comments:

- stars are sensitive to failures of hub, or links
- robust to a failure of hub, or single link

Fully connected



description: every node directly connected to every other node (also called a clique).

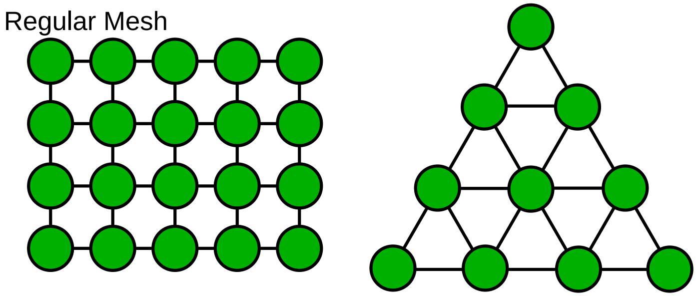
example:

- frame relay network (at a logical level)

comments:

- very robust to failures

Mesh



description:

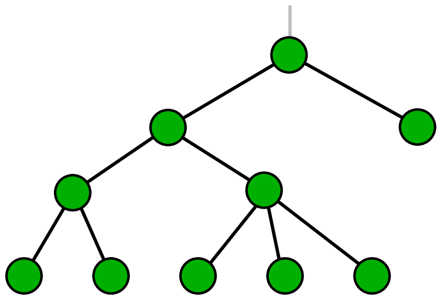
example:

- many real networks are somewhat meshy

comments:

- somewhere between clique, and star
- robust to failures

Tree



description: nodes are arranged as a tree (no loops)

examples:

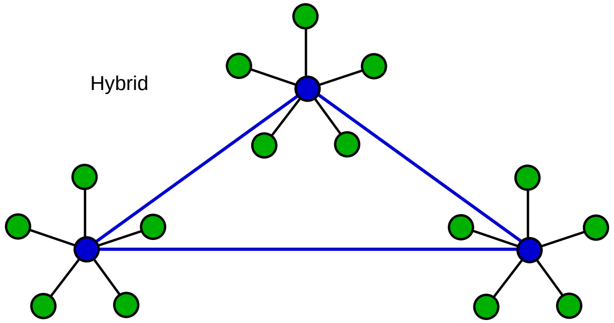
- shortest path trees in routing
- spanning tree protocol (for switched Ethernets)

comments:

- sensitive to failures

Very common for LANs, e.g. Ethernet

Hybrid



description: A combination of any two or more network topologies in such a way that the resulting network does not have one of the standard forms.
comments:

- a tree connected to a tree is still a tree network
- example is a hierachical network (as above)

Notation recap

Mostly as before (lecture 6)

- A **network** is a graph $G(N,E)$, with **nodes** $N = \{1,2,\dots n\}$ and **links** $E \subseteq N \times N$
- Offered traffic between O-D pair (p,q) is t_{pq}
- The set of all **paths** in $G(N,E)$ is $P = \cup_{[p,q]\in K} P_{pq}$
- Each link $e \in E$ has
  - a **capacity**, denoted by $r_e (\geq 0)$
  - a **distance** $d_e (\geq 0)$
  - a **load** $f_e (\geq 0)$
- The vector $\mathbf{x} = (x_\mu : \mu \in P)$ is called the **routing**

$$f_e = \sum_{\mu \in P : e \in \mu} x_\mu$$

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Primitive network design

- assume network nodes and edges are given
$$G = (N, E)$$
- find optimal routing $\mathbf{x}$, ignoring capacity constraints

Formulation: minimize $C(\mathbf{f})$ s.t.

$$\begin{aligned} f_e &= \sum_{\mu \in P : e \in \mu} x_\mu, & \forall e \in E \\ x_\mu &\geq 0, & \forall \mu \in P \\ \sum_{\mu \in P_{pq}} x_\mu &= t_{pq}, & \forall [p,q] \in K \end{aligned}$$

- use loads given by routing to obtain capacities, e.g.
$$r_e = f_e, \ \forall e \in E$$

More generally

- only network nodes are given
- we must decide edges as well as nodes
- routing is part of this
  - often assume shortest (physical) path routing
- in other design problems, even the nodes aren't given
  - e.g. cellular mobile phone network
  - we are not considering these cases in this course
- costs include
  - construction costs based on capacities r_e
  - performance costs (e.g. delays, reliability, ...) based on r_e and f_e

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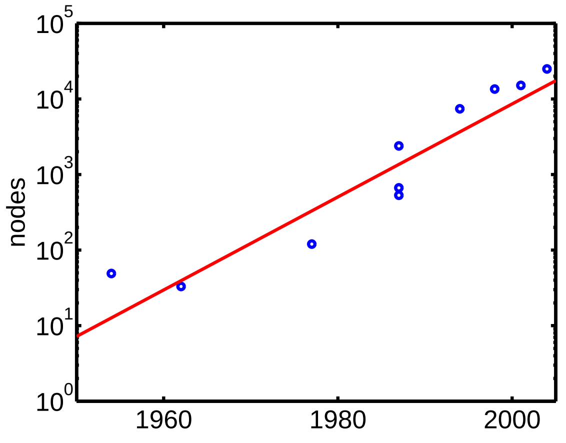
Minimal cost ring

- minimum cost path that visits each node exactly once, and returns to the start
- consider case where cost is linear in distance
  - minimum cost ring is the shortest ring
  - traveling salesman problem [1, 2, 3]
    - find the shortest tour between N nodes
    - e.g. a travelling salesman has to visit N cities (exactly once each), with the minimum travel distance, and return to his start point.
  - NP-complete or NP-hard (Non-Polynomial)
    - settle P versus NP problem and fetch a $1,000,000 prize

Travelling Salesman Computations

largest solvable problem has doubled in ~5 years [4]



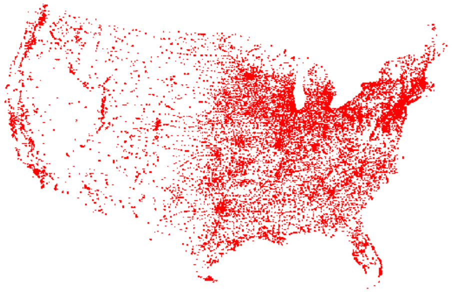
Current, can do $\sim 20,000$ nodes which is big enough for most networks, but not fast, or easy.

More about the travelling salesman problem can be found at
`http://www.tsp.gatech.edu/`

For the related Hamiltonian cycle problem (finding the shortest tour on a graph) see
`http://mathworld.wolfram.com/HamiltonianCircuit.html`

Travelling Salesman Example

13,509 nodes



http://www.tsp.gatech.edu/gallery/idata/usa13509.html

Minimal cost star

- all we need to do is choose the hub
- assume cost are linear in distances
- either compute or are given the distances between each pair of nodes
- simple calculate all column (or row) sums, and find the minimum
  - this gives the hub
  - only one routing is possible
  - compute capacities as for primitive case above
- complexity $O(N^2)$ which is pretty good
  - compared to NP-hard

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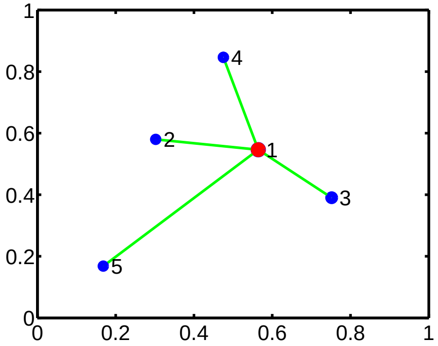
Minimal cost star: example



distances

	0.00	0.26	0.24	0.31	0.55
	0.26	0.00	0.49	0.32	0.43
	0.24	0.49	0.00	0.53	0.62
	0.31	0.32	0.53	0.00	0.74
	0.55	0.43	0.62	0.74	0.00
sums	1.37	1.50	1.89	1.91	2.35

Minimal cost star

- stars are used a lot
  - particularly at layer-2
    - Ethernets commonly use stars (at some level)
      - put stars together to get a tree
  - good where traffic matrix is not known
    - see later for why
- note often dual stars for reliability
  - backup star may be passive or active
    - active = load sharing
- not just used in comm.s networks
  - hub airports in the US

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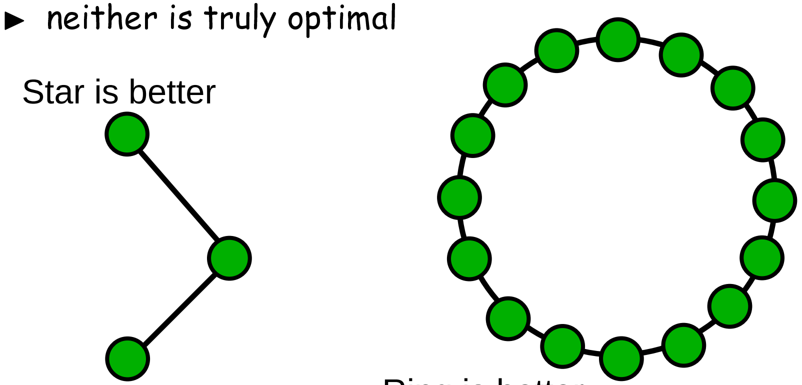
Which is better

- both very simple (conceptually)
- very different computationally
- a star or a ring can be better in some cases
- neither is truly optimal



References

[1] A. Schrijver, "On the history of combinatorial optimization (till 1960)." http://homepages.cwi.nl/~lex/.

[2] "Traveling salesman problem." http://www.tsp.gatech.edu/index.html.

[3] D. Applegate, R. Bixby, V. Chvátal, and W. Cook, Computational Combinatorial Optimization, ch. TSP cuts which do not conform to the template paradigm, pp. 261–304. Springer, 2001.

[4] "Milestones in the solution of TSP instances." http://www.tsp.gatech.edu/history/milestone.html.

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