Communications Network Design

lecture 07

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Routing (continued)

We continue the algorithmic viewpoint by considering an alternative to Dijkstra called the Floyd-Warshall algorithm. Also we consider routing implementation: OSPF, IS-IS, and some miscellaneous issues such as load balancing. Finally we will look into the distributed Bellman-Ford dynamic programming algorithm as implemented in RIP.

Floyd-Warshall

Alternative to Dijkstra for **all-pairs** shortest path problem

- same input as Dijkstra (except no start node)
- add nodes in one by one, and compute shortest paths as you add in a node
  - shortest path is either the same
  - or changes to include the new node

Floyd-Warshall

Let $D_{ij}^{(k)}$ denote the shortest path length from node i to node j using intermediate nodes from 1 to k only.

Initialise: $D_{ij}^{(0)} = d_{ij} \quad \forall i, j \in N$

$V^{(0)} = [0]$, an $|N| \times |N|$ zero matrix.

Step: for $k = 1, 2, \ldots n$, compute new distance estimates

$$D_{ij}^{(k)} = \text{min}\{D_{ij}^{(k-1)}, D_{ik}^{(k-1)} + D_{kj}^{(k-1)}\} \quad \forall\ i \neq j$$

Compute the predecessor nodes

If $D_{ij}^{(k)} < D_{ij}^{(k-1)}$ put $V_{ij}^{(k)} = k$;

otherwise, $V_{ij}^{(k)} = V_{ij}^{(k-1)}$

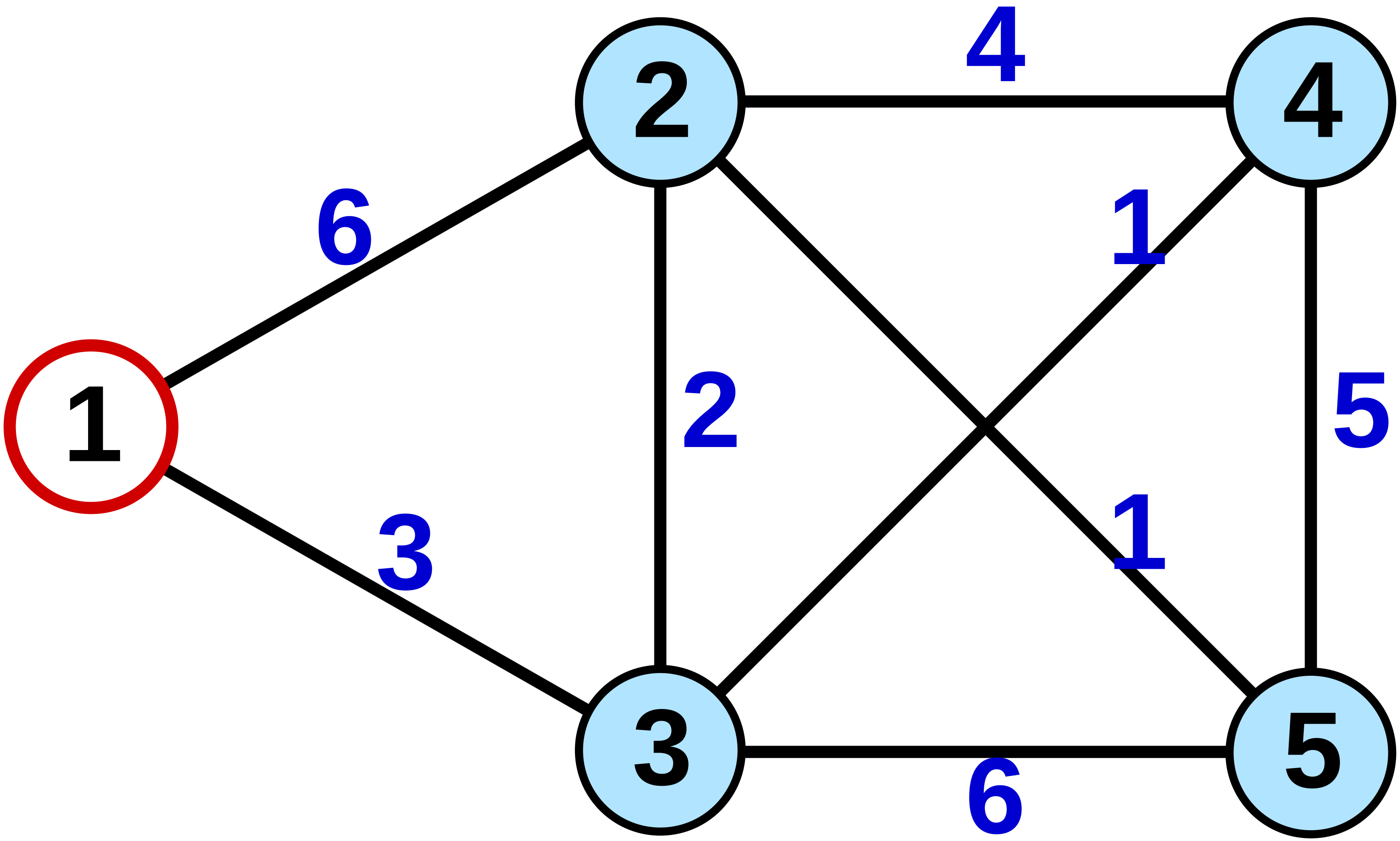
Floyd-Warshall

- The initialisation step gives the shortest path lengths subject to no intermediate nodes
- For a given k, $D_{ij}^{(k-1)}$ gives the shortest path from i to j using only nodes 1 through $k-1$ as possible intermediate nodes.
- On allowing node k as an intermediate node, either k IS on the shortest path, or it isn't.
  - **it isn't:** keep the same distance, and path
    - $D_{ij}^{(k)} = D_{ij}^{(k-1)}$ and $V_{ij}^{(k)} = V_{ij}^{(k-1)}$
  - **it is:** the new path must be made of two shortest paths, joined by node k, i.e. i–k and k–j
    - $D_{ij}^{(k)} = D_{ik}^{(k-1)} + D_{kj}^{(k-1)}$
    - $V_{ij}^{(k)}$ shows where the join occurred

Floyd-Warshall

- The 0's in $V^{(n)}$ determine the adjacencies (links) in the final network.
  - $V_{ij}^{(n)}$ indicates that we never found a shorter path than d_{ij} along the direct path.
  - hence i and j are adjacent in the SPF tree
- The other terms in $V^{(n)}$ show the predecessor nodes for each end-to-end path.
  - construct paths, by concatenating predecessor nodes

Floyd-Warshall example



Floyd-Warshall example

Initially, we put direct links into the matrix D

$$
D_{ij}^{(0)} = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 6 & 3 & \infty & \infty \\
2 & & 0 & 2 & 4 & 1 \\
3 & & & 0 & 1 & 6 \\
4 & & & & 0 & 5 \\
5 & & & & & 0
\end{array}
\qquad
V^{(0)} = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 \\
2 & & 0 & 0 & 0 & 0 \\
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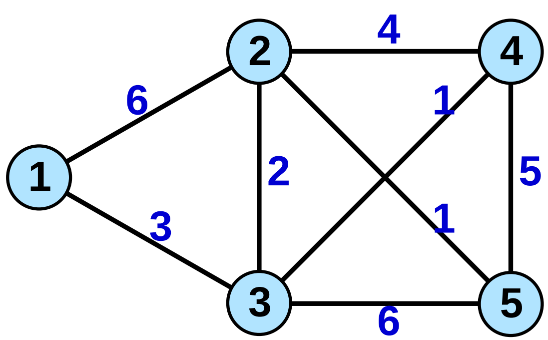
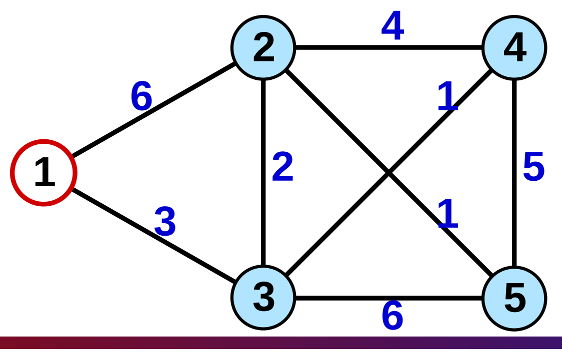
\end{array}
$$



Floyd-Warshall example

k = 1: include node 1 on existing direct paths (so any path already containing node 1 e.g. top line and first column of D, can be ignored). Here, nothing changes.

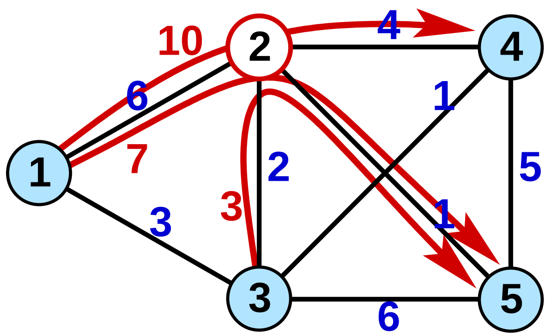
$$D_{ij}^{(1)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 6 & 3 & \infty & \infty \\ 2 & & 0 & 2 & 4 & 1 \\ 3 & & & 0 & 1 & 6 \\ 4 & & & & 0 & 5 \\ 5 & & & & & 0 \end{array} \qquad V^{(1)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & & 0 & 0 & 0 & 0 \\ 3 & & & 0 & 0 & 0 \\ 4 & & & & 0 & 0 \\ 5 & & & & & 0 \end{array}$$



Floyd-Warshall example

k = 2: try including node 2 on existing paths (so any path already containing node 2 e.g. line 2 and second column of D, can be ignored).

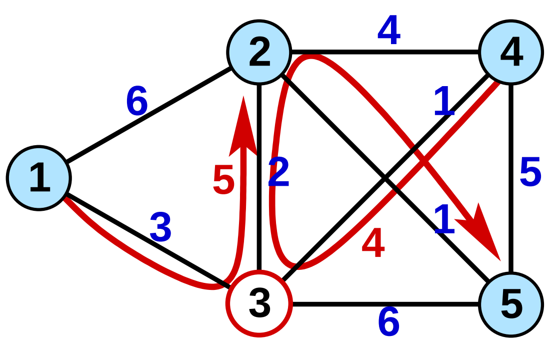
$$D_{ij}^{(2)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 6 & 3 & \mathbf{10} & \mathbf{7} \\ 2 & & 0 & 2 & 4 & 1 \\ 3 & & & 0 & 1 & \mathbf{3} \\ 4 & & & & 0 & 5 \\ 5 & & & & & 0 \end{array} \qquad V^{(2)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 0 & \mathbf{2} & \mathbf{2} \\ 2 & & 0 & 0 & 0 & 0 \\ 3 & & & 0 & 0 & \mathbf{2} \\ 4 & & & & 0 & 0 \\ 5 & & & & & 0 \end{array}$$



Floyd-Warshall example

$\mathbf{k = 3}$: try including node 3 on existing paths (so any path already containing node 3 e.g. line 3 and third column of D, can be ignored).

$$D_{ij}^{(3)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \mathbf{5} & 3 & \mathbf{4} & \mathbf{6} \\ 2 & & 0 & 2 & \mathbf{3} & 1 \\ 3 & & & 0 & 1 & 3 \\ 4 & & & & 0 & \mathbf{4} \\ 5 & & & & & 0 \end{array} \qquad V^{(3)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \mathbf{3} & 0 & \mathbf{3} & \mathbf{3} \\ 2 & & 0 & 0 & \mathbf{3} & 0 \\ 3 & & & 0 & 0 & 2 \\ 4 & & & & 0 & \mathbf{3} \\ 5 & & & & & 0 \end{array}$$



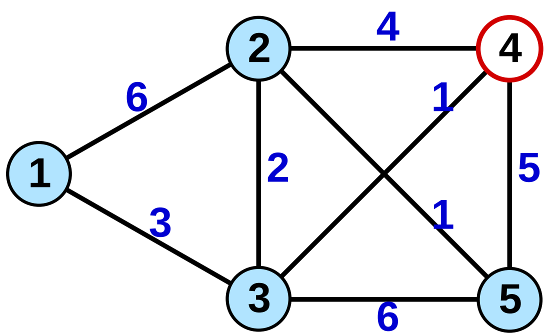
E.G. The old path joining 4-5 was a direct link with distance $D_{45}^{(2)} = 5$. But when we are allowed to include node 3, we get an alternative $D_{43}^{(2)} + D_{35}^{(2)} = 4$, which is better, so we set $D_{45}^{(3)} = 4$, and $V_{45}^{(3)} = 3$.

Floyd-Warshall example

$\mathbf{k = 4}$: try including node 4 on existing paths:
No changes.

$$D_{ij}^{(4)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 5 & 3 & 4 & 6 \\ 2 & & 0 & 2 & 3 & 1 \\ 3 & & & 0 & 1 & 3 \\ 4 & & & & 0 & 4 \\ 5 & & & & & 0 \end{array}$$

$$V^{(4)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 0 & 3 & 3 \\ 2 & & 0 & 0 & 3 & 0 \\ 3 & & & 0 & 0 & 2 \\ 4 & & & & 0 & 3 \\ 5 & & & & & 0 \end{array}$$



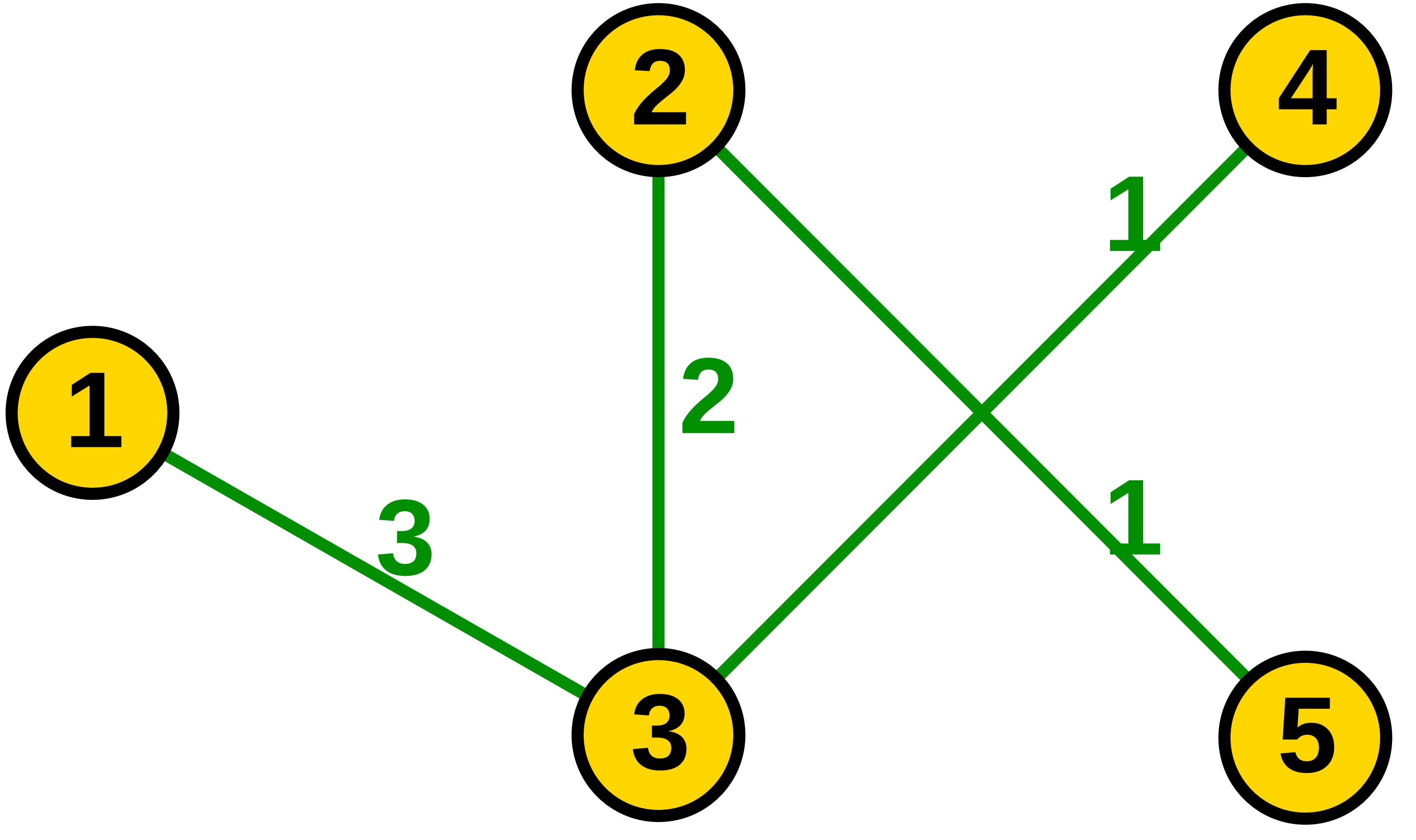
Floyd-Warshall example

$\mathbf{k = 5}$: try including node 5 on existing paths. The entries $D_{ij}^{(5)}$ give the length of the shortest path from each node i to each other node j.

$$D_{ij}^{(5)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 5 & 3 & 4 & 6 \\ 2 & & 0 & 2 & 3 & 1 \\ 3 & & & 0 & 1 & 3 \\ 4 & & & & 0 & 4 \\ 5 & & & & & 0 \end{array} \qquad V^{(5)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & \boxed{0} & 3 & 3 \\ 2 & & 0 & \boxed{0} & 3 & \boxed{0} \\ 3 & & & 0 & \boxed{0} & 2 \\ 4 & & & & 0 & 3 \\ 5 & & & & & 0 \end{array}$$

Use the boxed zero entries in the final V to determine links: (1,3), (2,3), (2,5), (3,4).

Floyd-Warshall shortest paths



Floyd-Warshall complexity

- In calculating $D_{ij}^{(k)}$ at each step, we need to compare two possibilities for each of $\frac{|N|(|N|-1)}{2}$ pairs of nodes.
- the algorithm has $|N|$ steps
- total computational complexity is $O(|N|^3)$.
- This of course is the same as repeating simple version of Dijkstra's algorithm $|N|$ times (for each of $|N|$ sources)

Alternative algorithms

- Dijkstra and FW assume non-negative weights
- not a problem for network applications
- for more general applications, use Bellman-Ford
  - can be used on graphs with negative edge weights
  - as long as the graph contains no negative cycle reachable from the source node
- Johnson's algorithm solves all pairs shortest paths, may be faster than Floyd-Warshall on sparse graphs.

Routing implementation

- must obtain **consistent** results between routers
  - to avoid route loops, or dead-ends
- must **adapt** to changing network
  - route around link or node failures
- must use a **distributed** algorithm
  - an algorithm which enables a common objective of two or more peer processes to be performed jointly by the combination of processing and exchanging information.
  - The distributed algorithm is broken down into a set of local algorithms, one of which is performed by each peer process.
  - Each local process carries out various operations on the available data, and at various points in the algorithm, it sends/receives data to/from other peer processes.

SPF implementation

Implementation is performed by a **routing protocol**

- routing protocol performs SPF calculation
- first needs to find out the topology, and weights
- each router **floods** its available topology information to all other routers
  - takes the form of LSAs
    - Link State Announcements
    - a router sends LSA describing its links to adjacent routers
      - LSA includes link weight
    - neighbours forward (non-duplicate) LSAs to their neighbours
  - hence this is called a **link-state** routing protocol

SPF implementation

- once a router has seen all LSA
  - it knows the complete topology
  - it can perform Dijkstra to compute shortest paths to all other routers
- note that each router only needs to perform Dijkstra once
  - it only needs to know paths from itself, to the other routers.
  - hence $O(|N|^2)$ for simple implementation
  - $O(|N|^3)$ workload is distributed over $|N|$ routers

SPF routing implementations

- common implementations
  - OSPF [1]
    - Open Shortest Path First
    - several RFCs needed to see all possibilities
  - IS-IS [2]
    - Intermediate System-Intermediate System
    - several RFCs needed to see all possibilities
- some amusement: RFC 4041, "Requirements for Morality Sections in Routing Area Drafts"
  `ftp://ftp.rfc-editor.org/in-notes/rfc4041.txt`

  It has often been the case that morality has not been given proper consideration in the design and specification of protocols produced within the Routing Area. This has led to a decline in the moral values within the Internet and attempts to retrofit a suitable moral code to implemented and deployed protocols has been shown to be sub-optimal...

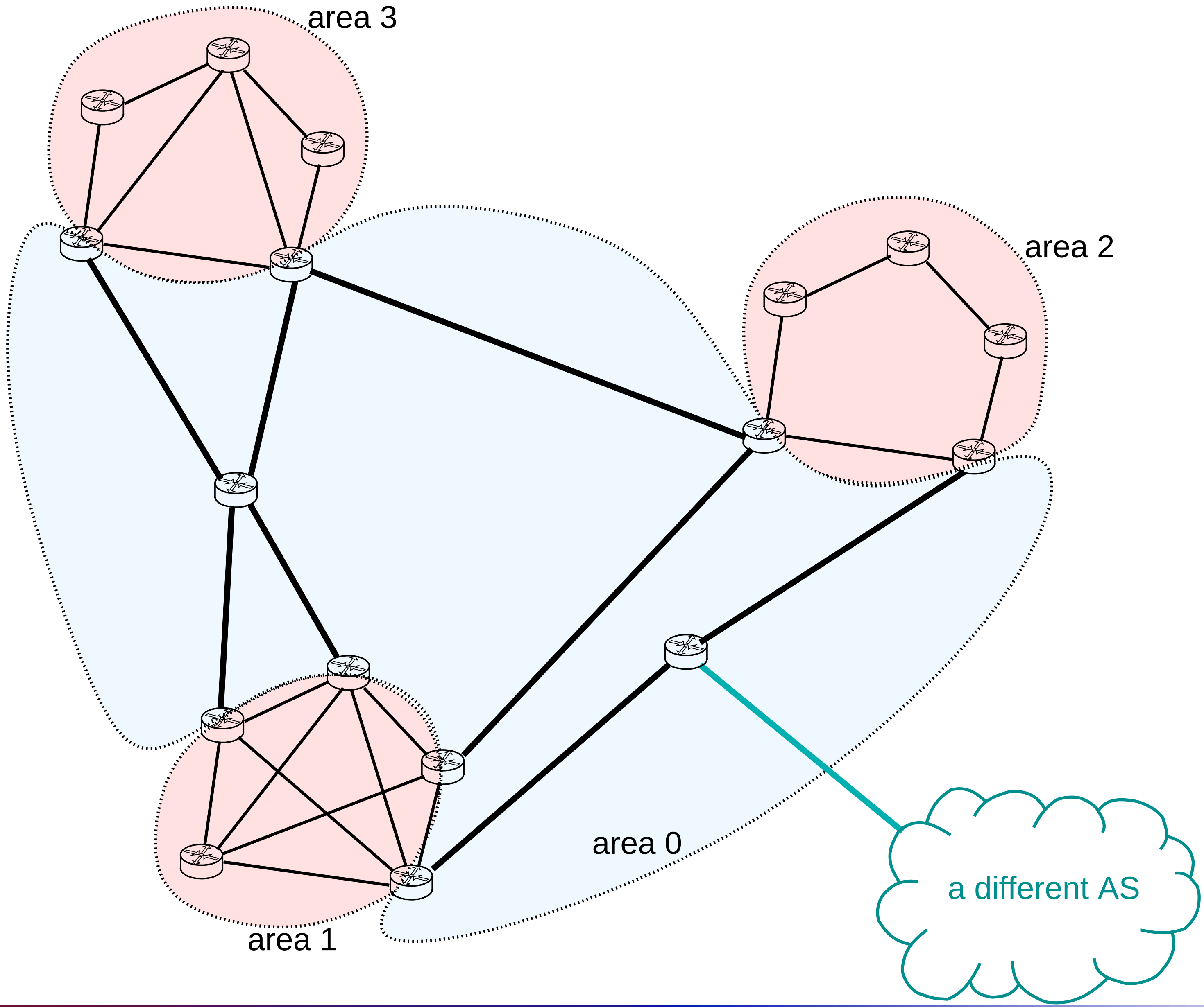
OSPF

- **soft state**
  - periodically refresh LSA information
  - also exchange `hello` messages (between neighbouring routers) to test link states
  - in case a failure happens, and isn't detected
- not routed
  - LSAs are just sent in IP packets
    - like everything else
  - transmitted over IP (protocol 89)
    - not over TCP, so not reliable transport
  - but you can't route, until you have routes
  - hence forwarding of LSAs is limited to **adjacent** routers

Scaling of OSPF

- as noted earlier, if $|N|$ is too large, computing SPF takes too long, and we run into problems
- how can you build large ($|N| \sim 1000$) networks
- use (2 level) hierachy
  - in subnetworks compute shortest paths
  - compute the shortest paths between subnets
  - combine the two
- not as simple as it sounds
  - example OSPF **areas**
  - `area 0` is the backbone (1st level)
  - other `areas` are the subnetworks (2nd level)

Scaling of OSPF



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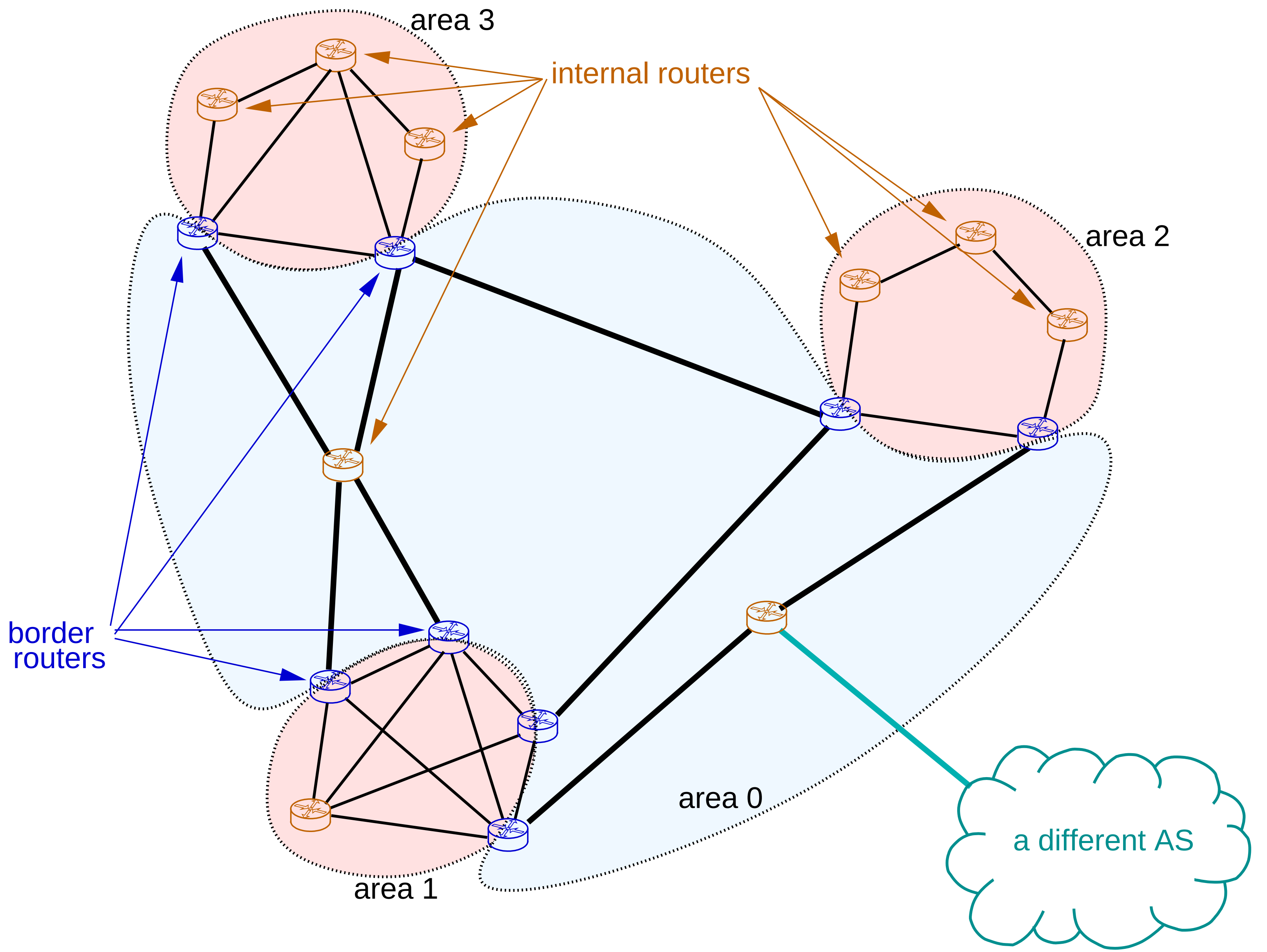

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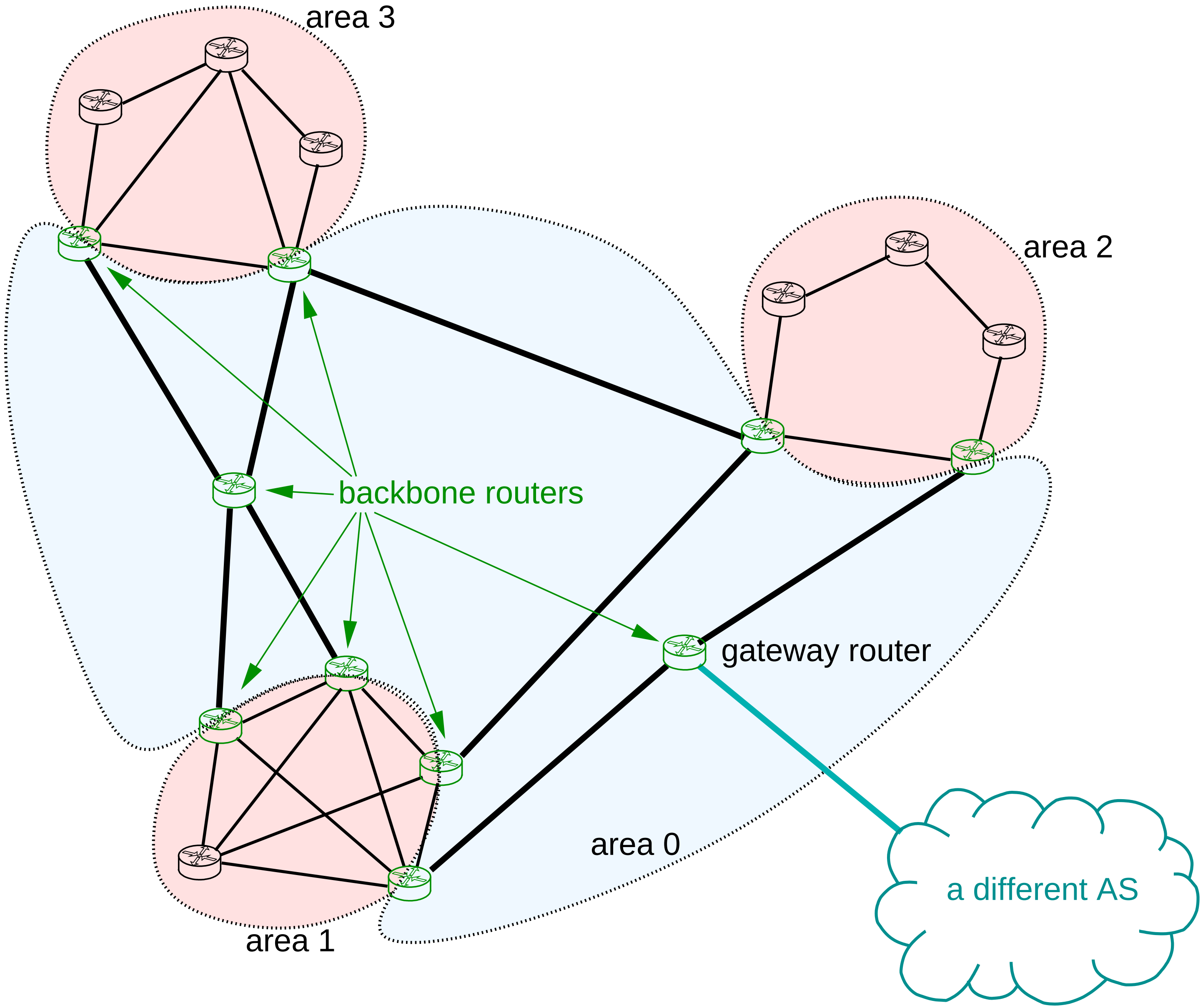
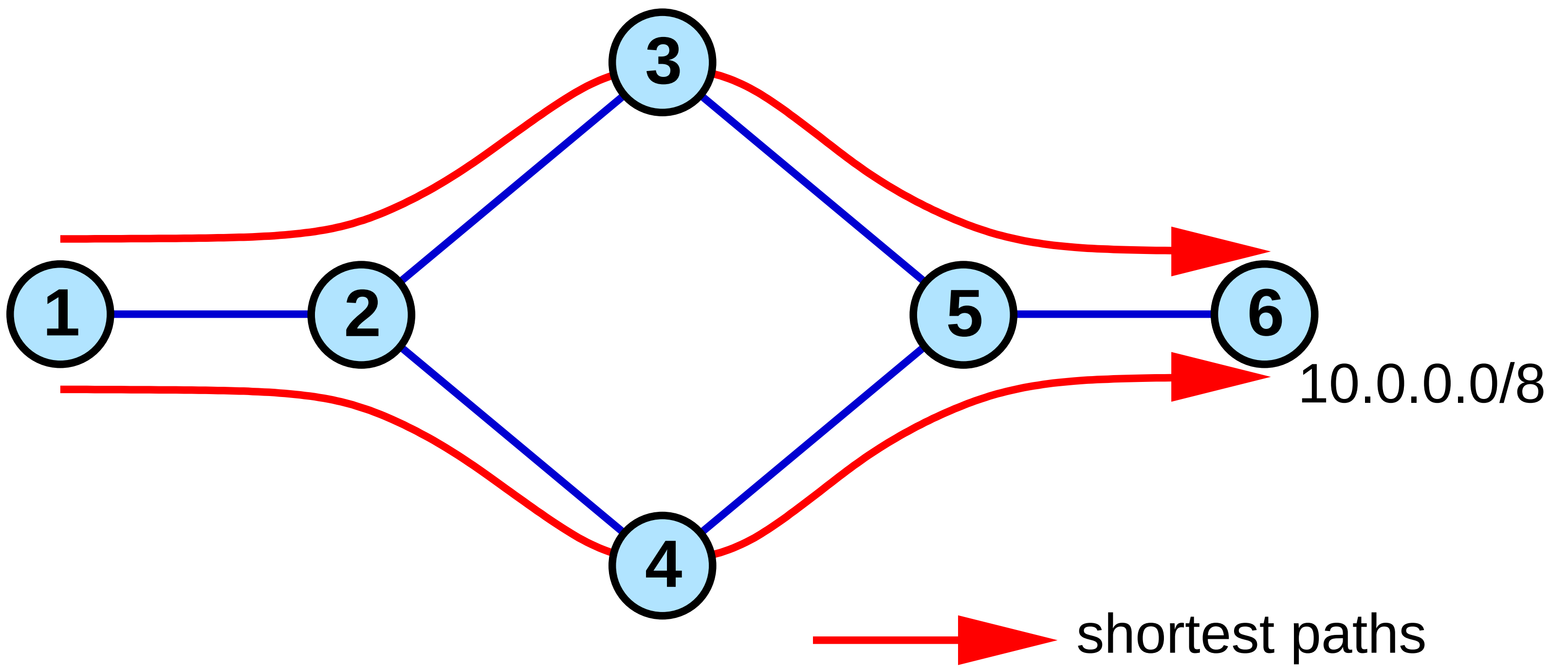


Load balancing

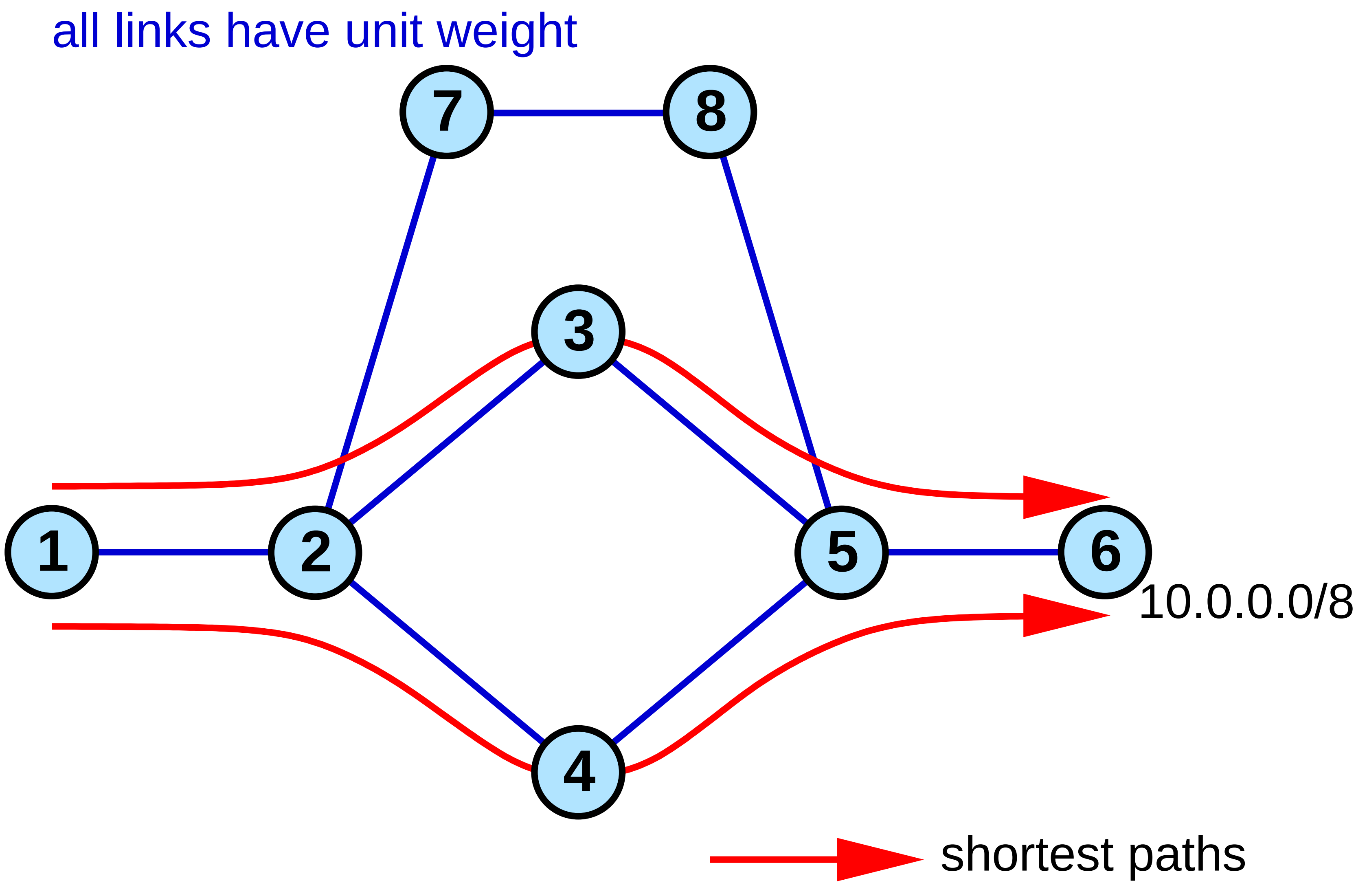
- in some cases there will be two (or more) equal distance paths from source to destination
- Dijkstra and FW only give you one path
- solution is non-unique
- more efficient to share load over both paths



Dijkstra and load balancing

- for all destination nodes in graph, you have a shortest path
- start at a particular destination
- recursively descend through neighbours at the right distance back
- algorithm exponential in number of paths, but this is hopefully small

Dijkstra and load balancing ex.



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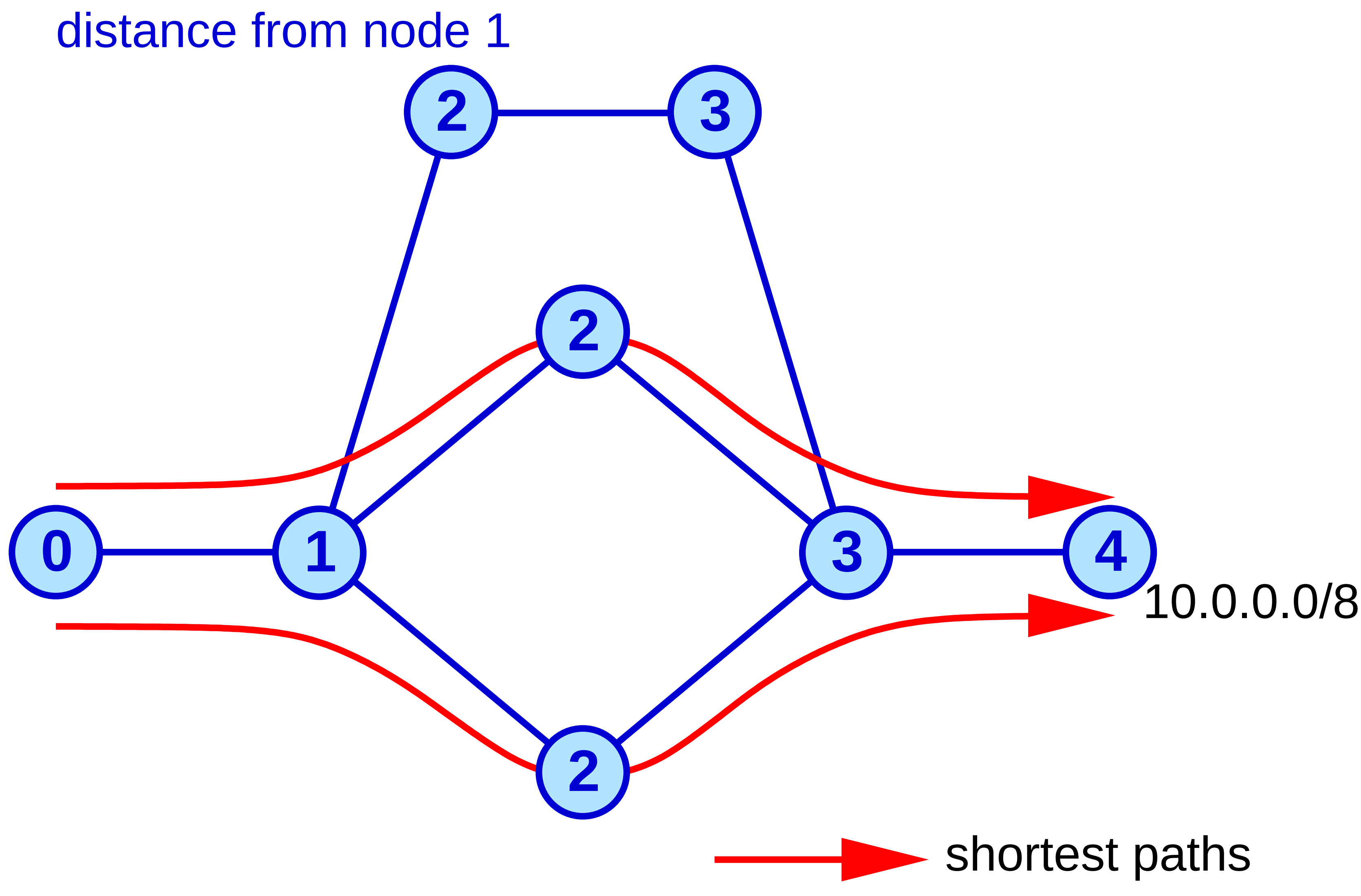

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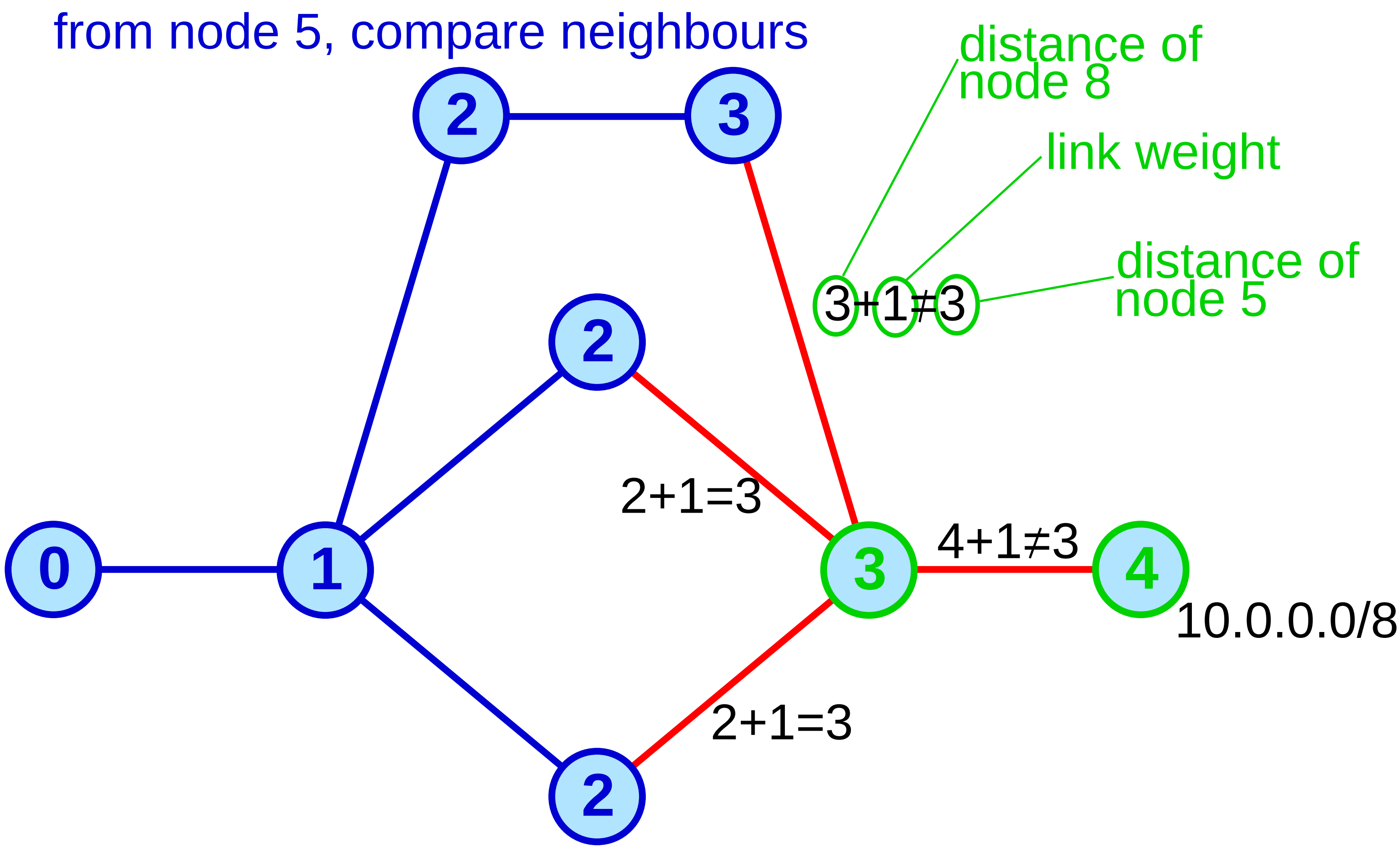

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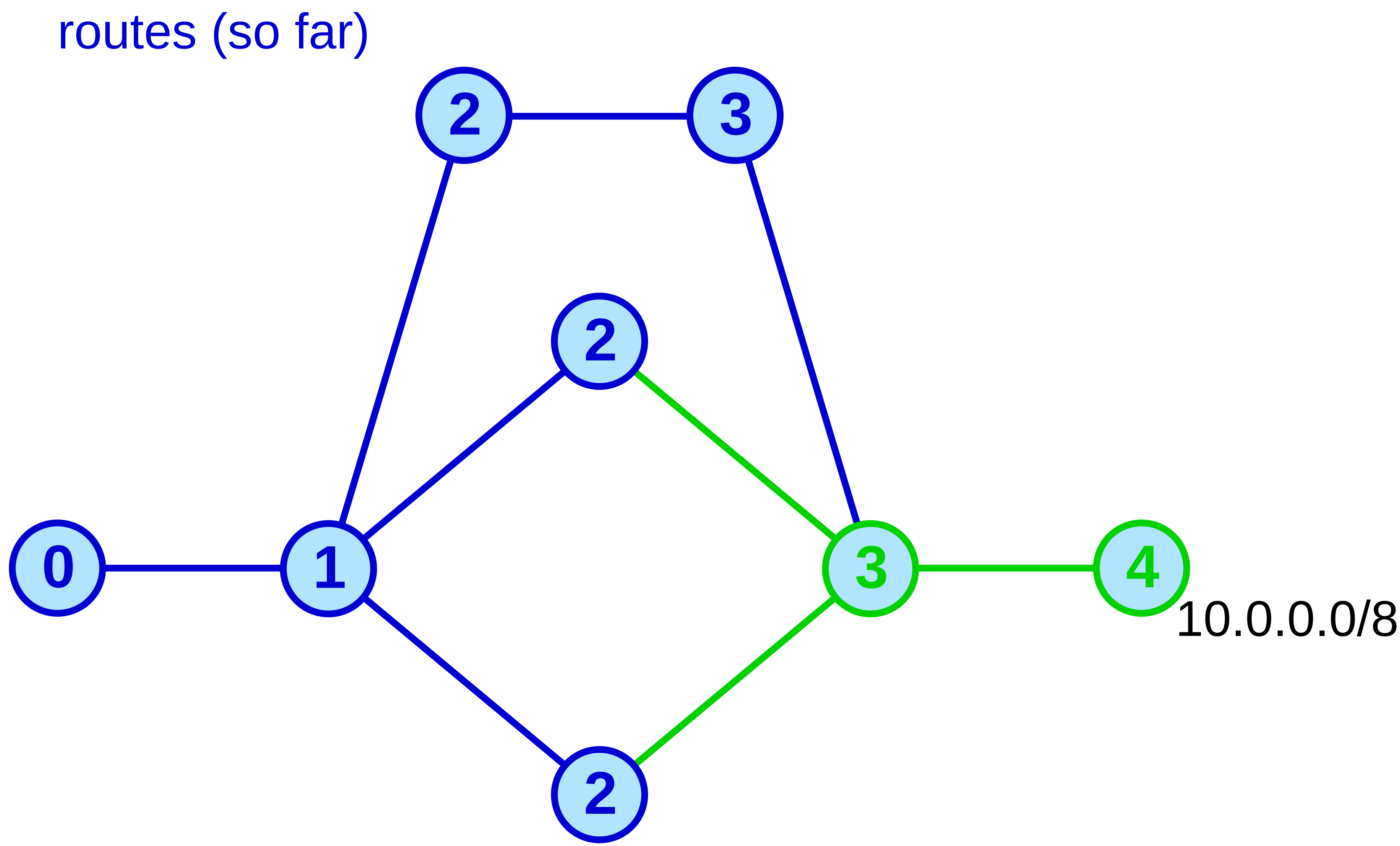


Load balancing implementation

- method one
  - split traffic up by addresses
  - instead of a simple forwarding table
    - e.g. at router 2, the next hop router to prefix 10.0.0.0/8 is router 3
  - have two forwarding table entries
  - e.g. forwarding table (at router 2)

destination	**next hop router**
10.0.0.0/9	3
10.1.0.0/9	4

- traffic betwen different prefixes may be uneven

Load balancing implementation

- method two
  - need multiple paths in forwarding table

destination	next hop router
10.0.0.0/8	3 or 4

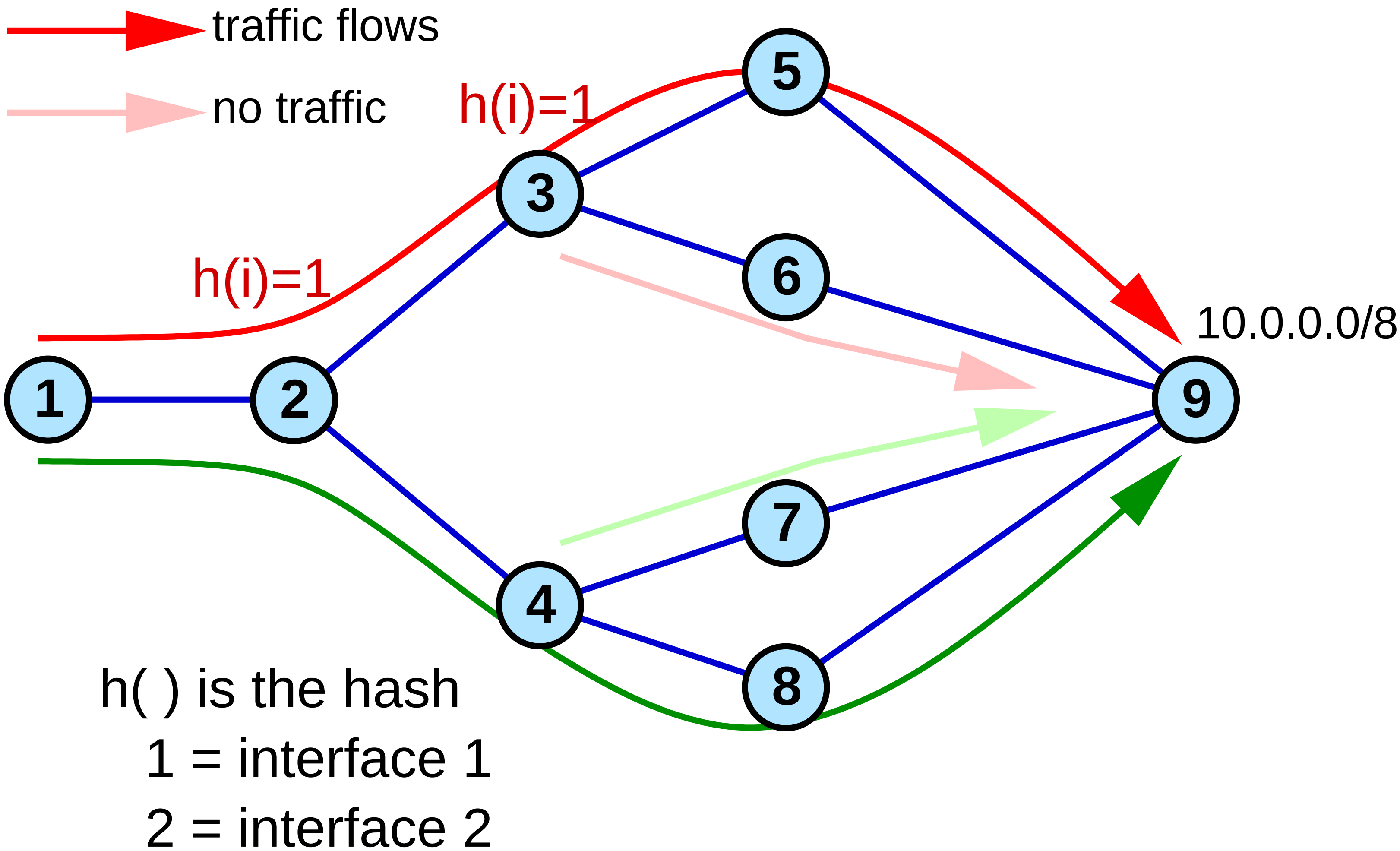
  - allocate traffic between two next hops randomly as it arrives
  - method is simpler to administrate
  - better balance of traffic
  - may **reorder** packets
- method two(b)
  - randomize first packet of a flow
  - subsequent packets of flow follow same route

Load balancing implementation

- method three
  - allocate traffic randomly between two paths
  - but randomization is based on a **hash** of the IP source and destination address
  - effect is random allocation
    - but with all packets between same source and destination using the same path
    - so no reordering within a TCP connection
  - hash needs to be randomized at each node, otherwise multiple splits don't work
    - different seeds for randomization at each router

Load balancing implementation

- method 3 without random seeds in hashes



Link weights

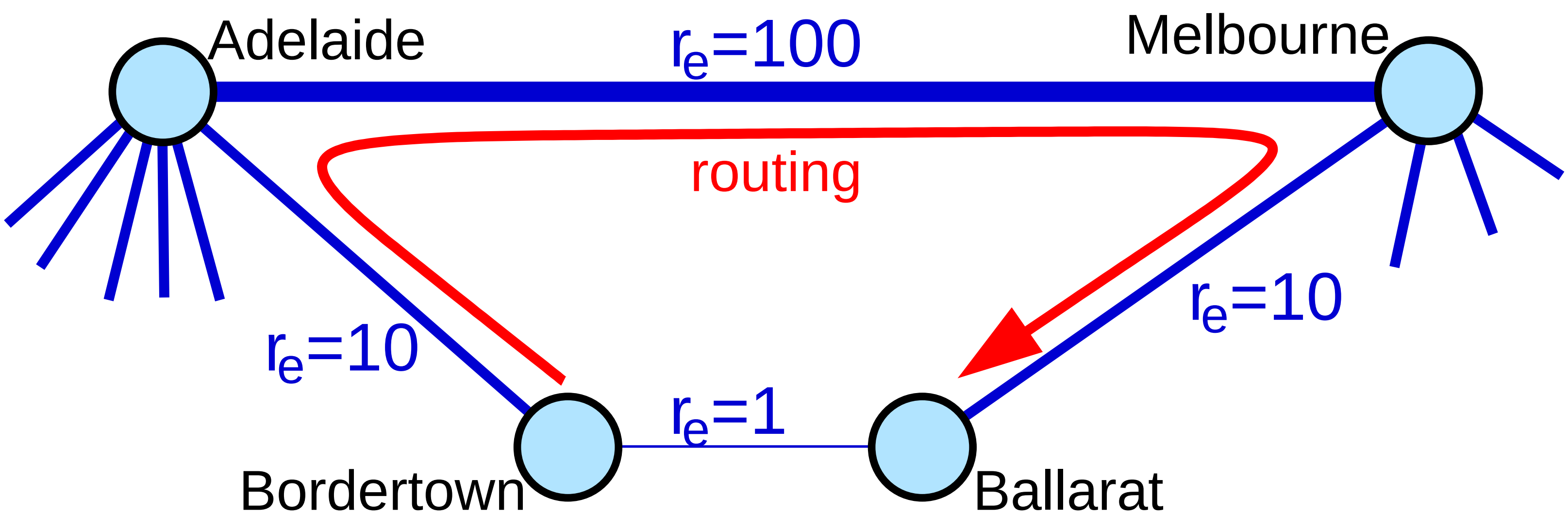
What should be the link weights α_e?

- real, physical distance?
- delay of packets along link?
- hop count (e.g. $\alpha_e = 1$)?
- some arbitrary number?

Cisco default

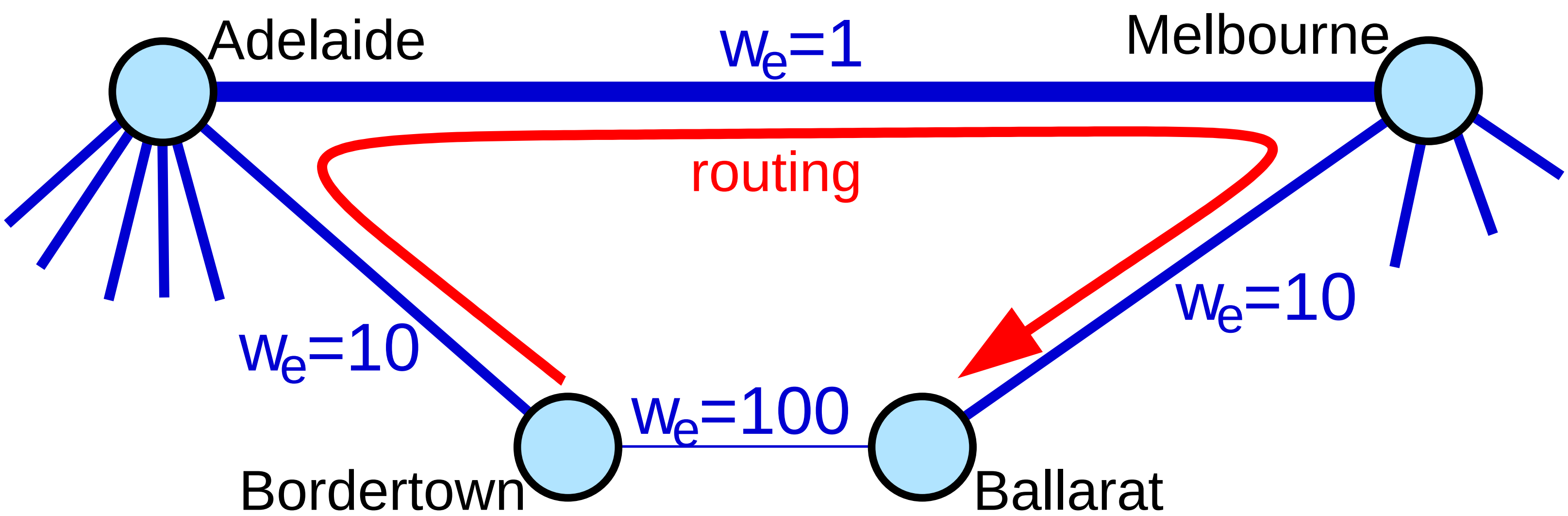
- inverse capacity weights $\alpha_e = A/r_e$
- the higher capacity links are nominally "shorter"
- encourages traffic to use higher capacity links
- it can lead to weird routing

Link weights



- may think don't need link between Ballarat and Bordertown, because it has no traffic
- but its just because routing is taking a longer path
  - direct path: $D = w_e = 100/r_e = 100$
  - indirect path: $D = 10 + 10 + 1 = 21$
- inverse capacity is often the wrong choice

Link weights



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Link weights

- correct choice depends on objectives
- common cases occur when minimizing delays:
  - if propagation delay is dominant
    - minimize physical path distance
    - weight = link distance, e.g. $\alpha_e = d_e$
  - if processing and transmission time dominate
    - minimize the hop count, e.g. $\alpha_e = 1$
  - if queueing causes most delays, need to minimize loads on links
    - early ARPANET had load-sensitive routing
    - measured packet delays along links (to get α_e)
    - sent packet along shortest (delay) path
- can also write link weight choice as an optimization problem (called **traffic engineering**)

Incremental Dijkstra

As noted above, Dijkstra doesn't scale as well as we might like.

- network of 1000 nodes need some kind of hierachy
- alternatively, note that most of the time the network doesn't change
  - when it does change, it is usually only a local change in a few links
  - perhaps we don't have to recompute everything from scratch?
- incremental Dijkstra algorithm
- latest implementations use incremental Dijkstra.

http://www.cisco.com/en/US/about/ac123/ac114/ac173/Q3-04/sp_calculate.html

Generalization

We focused here on IP routing

- but routing is needed in most communications networks

Shortest paths used in many areas – not just communications networks

- there are many other types of networks
  - often want shortest paths on these
  - e.g. for finding close linkages in social networks
- not always obvious what's a network
  - Dijkstra used in image processing
  - pixels form a grid, which is a network
- Dijkstra is often a component of another algorithm

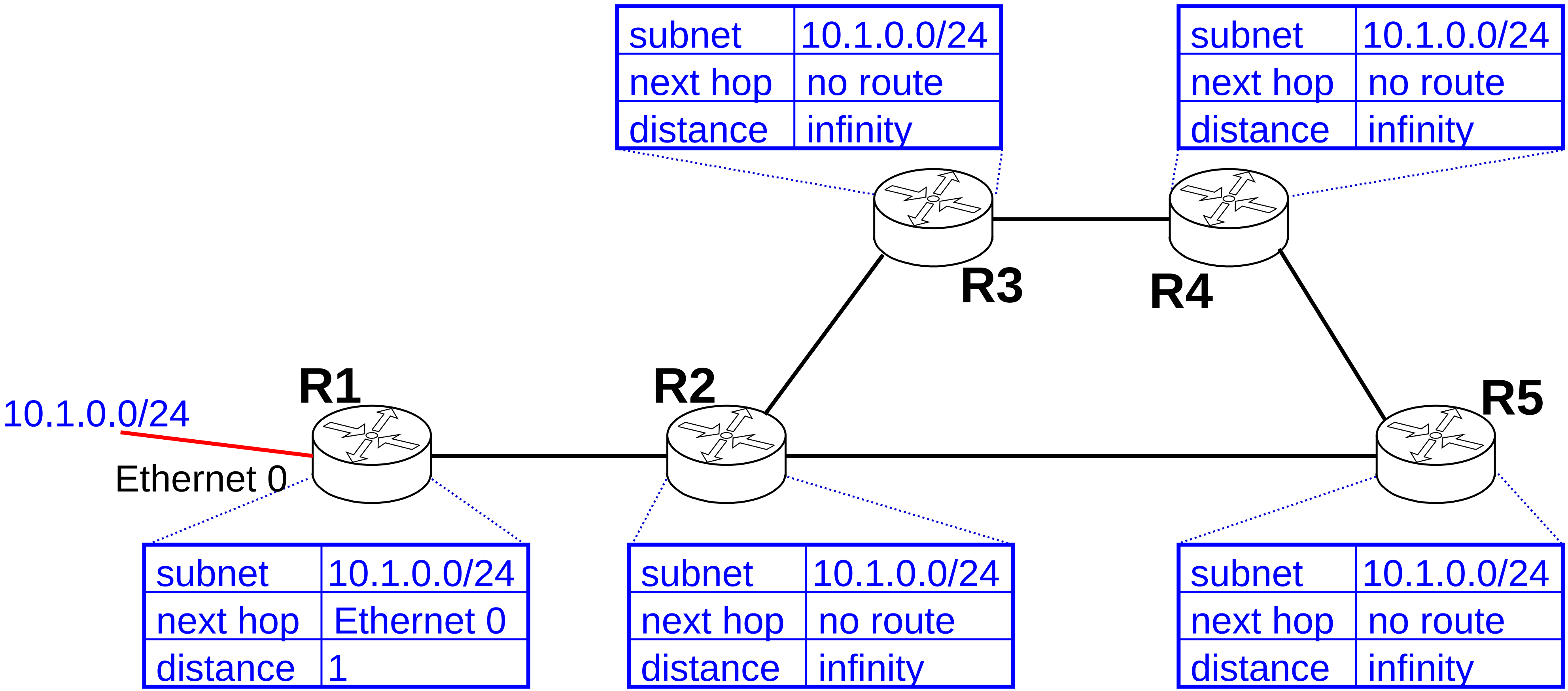
Link state vs Distance Vector

- We saw OSPF was a **link-state** routing protocol
  - floods topology (link states), and computes SPF
  - solves shortest path problem
- alternative is called **distance-vector** protocol
  - examples: RIP, IGRP, ...
  - originally also aimed to solve shortest paths
    - but nodes don't need to know complete topology
- hybrids exist, e.g. EIGRP

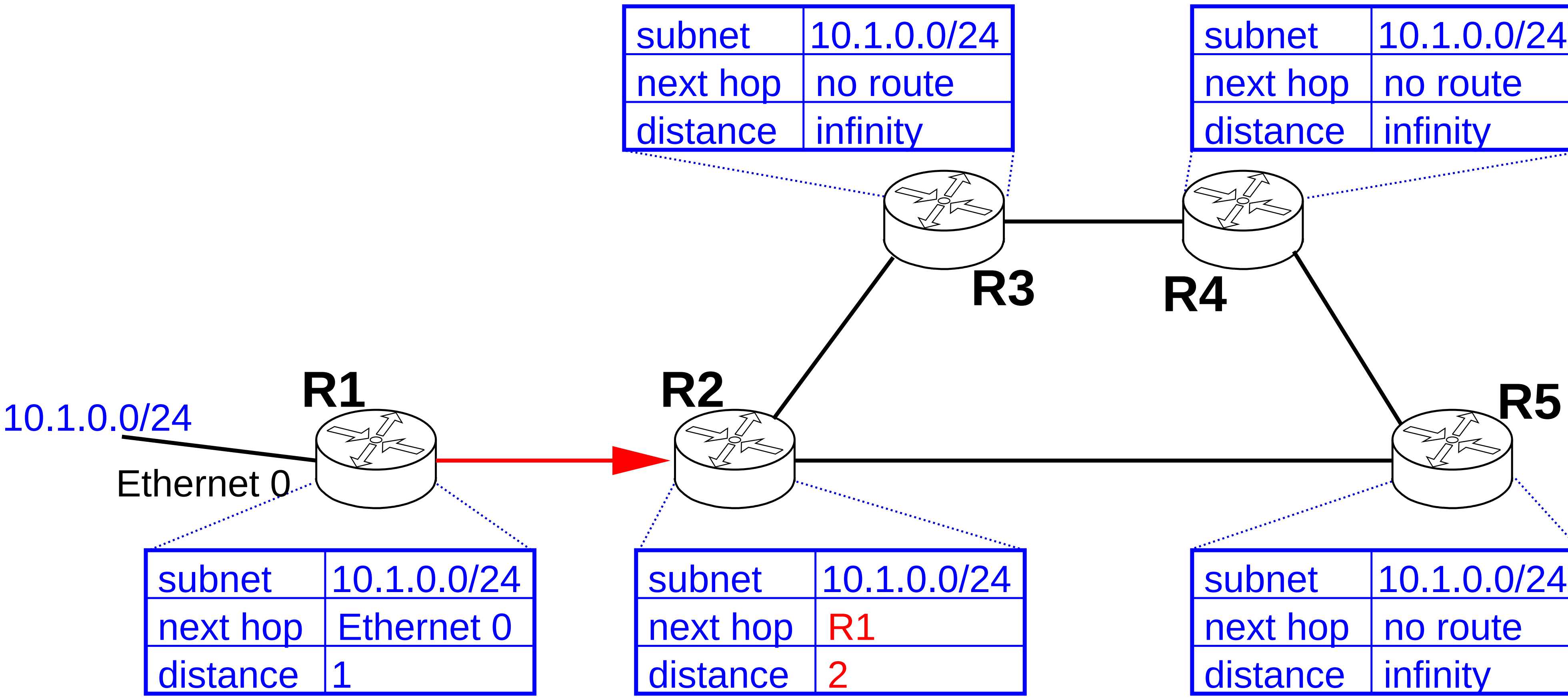
Distance Vector

- Make a list of destinations you can reach and the distance to these destinations.
  - Store in routing table
- Share this list with your neighbours
- Add to routing table new information gained from adjacent routers about the destinations they can reach
  - remember to increment their distance
  - keep the source as the next hop
- If two paths to the same destination exists, keep the shortest distance path.
- Repeat periodically (in RIP every 30 seconds).

Distance Vector example



Distance Vector example



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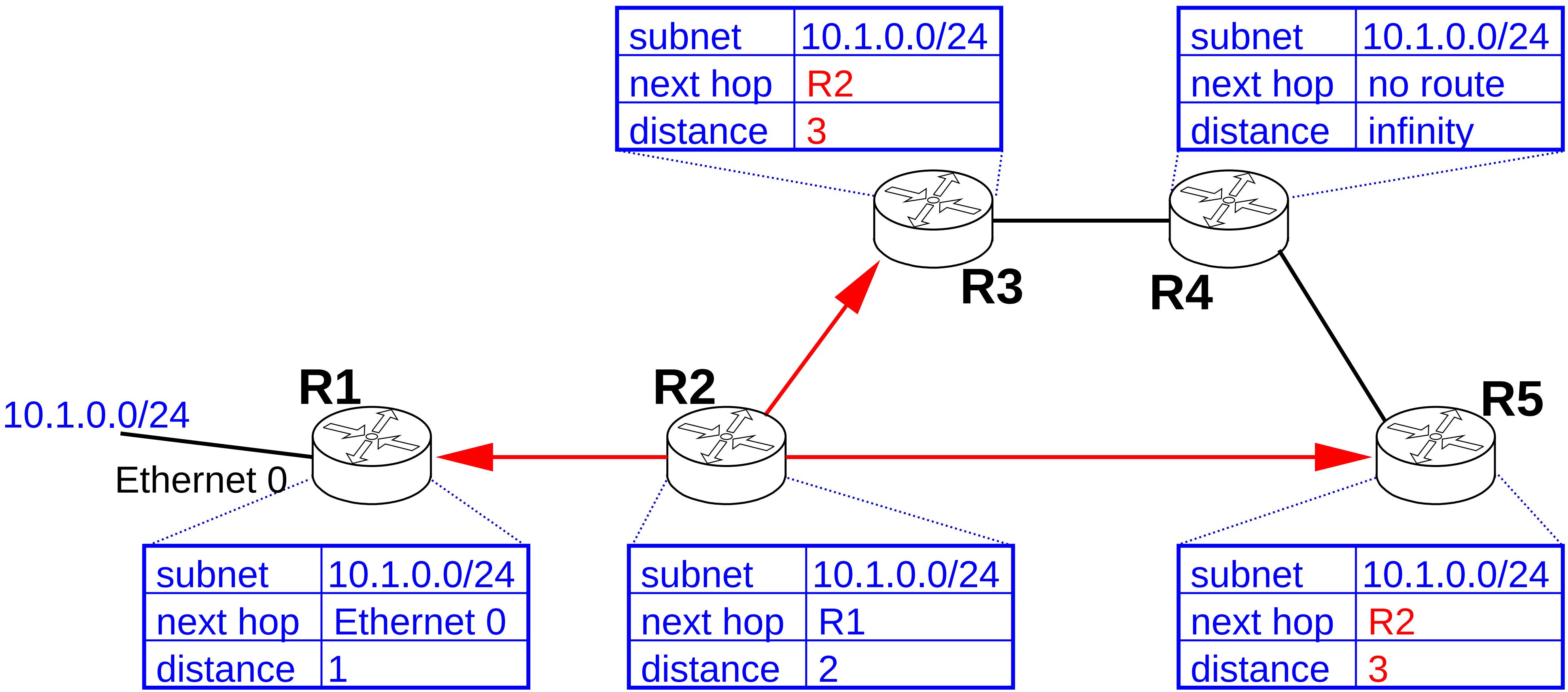
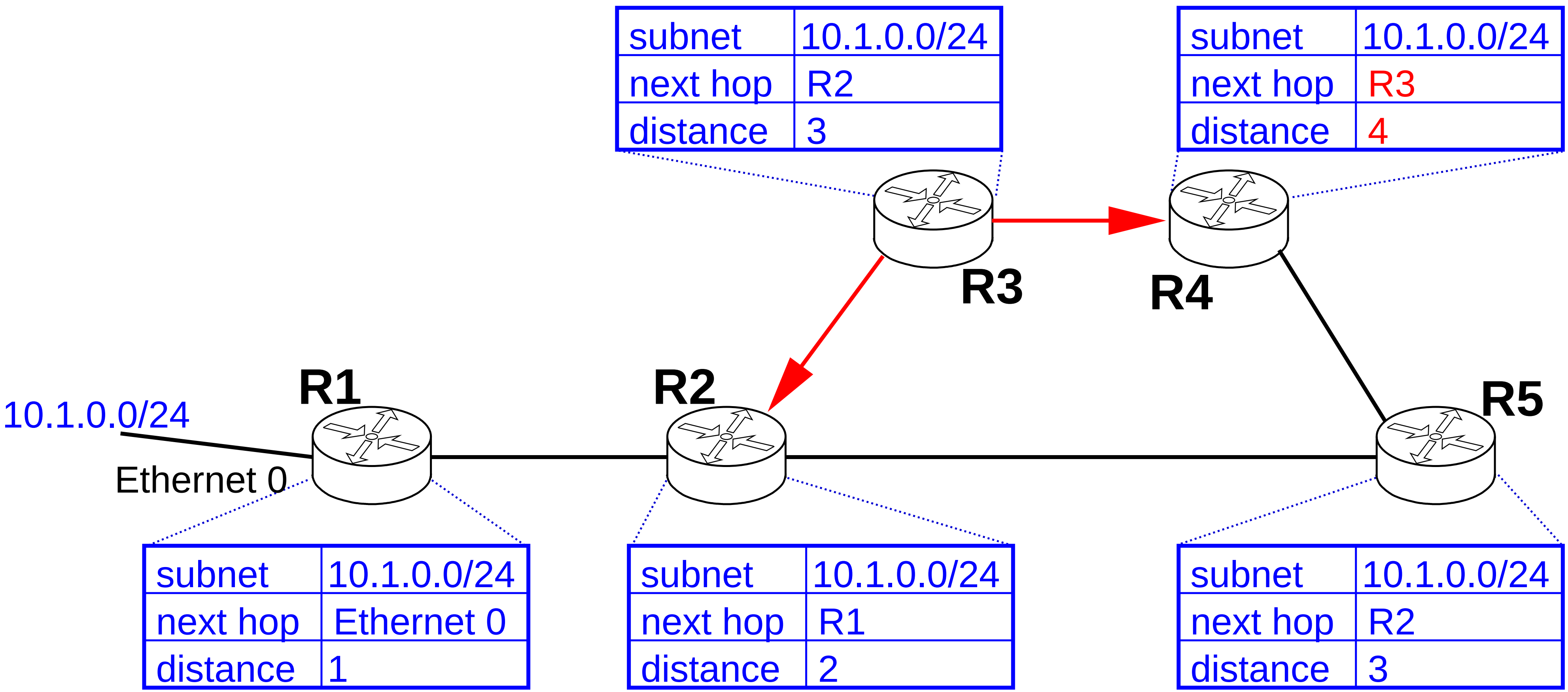
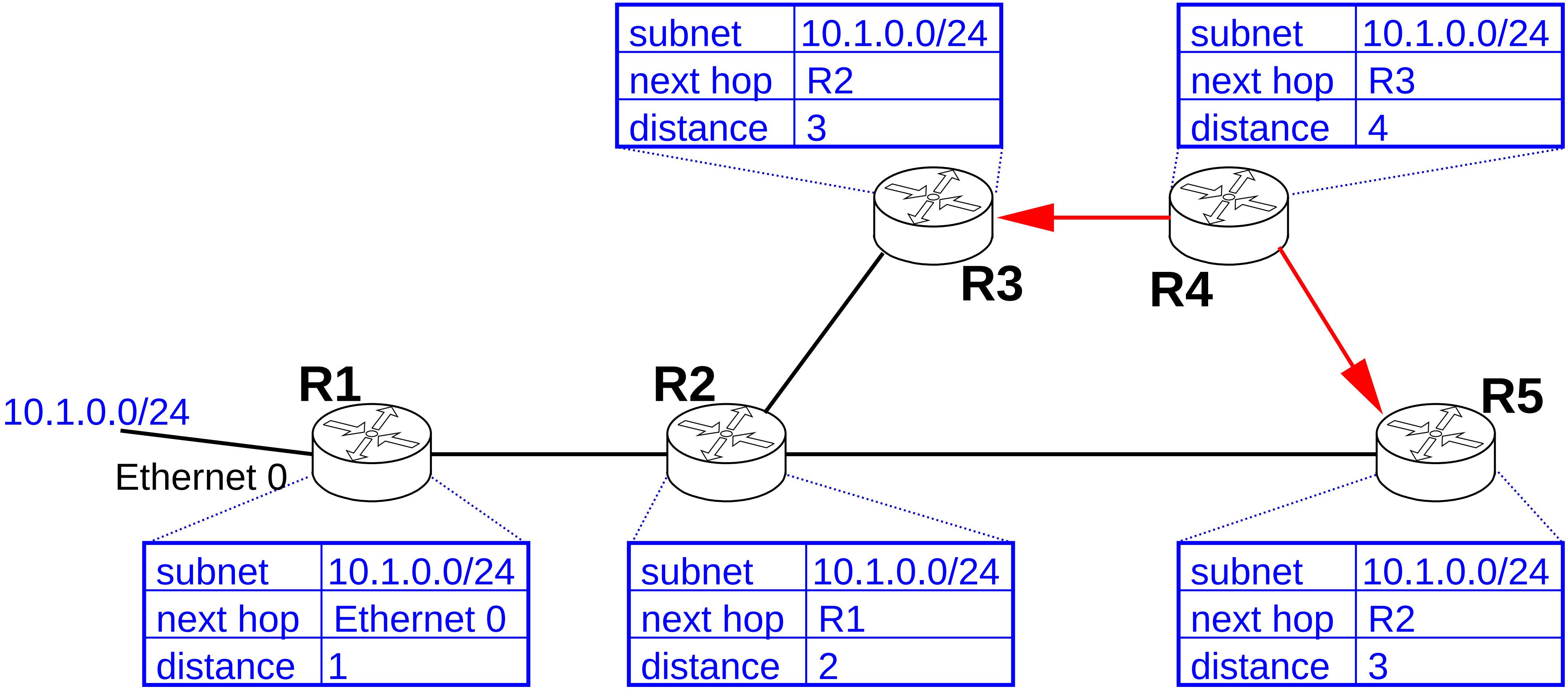


Distance Vector example



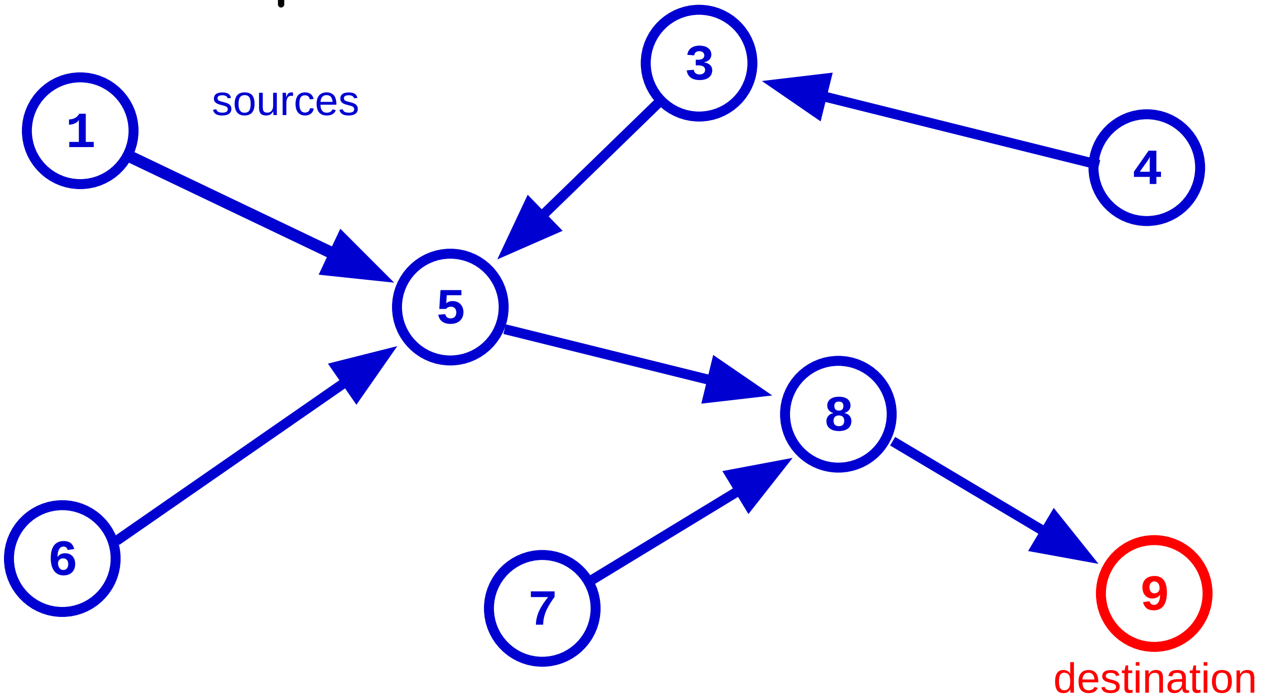
Distance Vector example



Sink trees

Results of algorithm must be a sink tree

- "sink" is destination
- get a tree leading to the destination
- must be a tree: shortest path can only be composed of shortest paths



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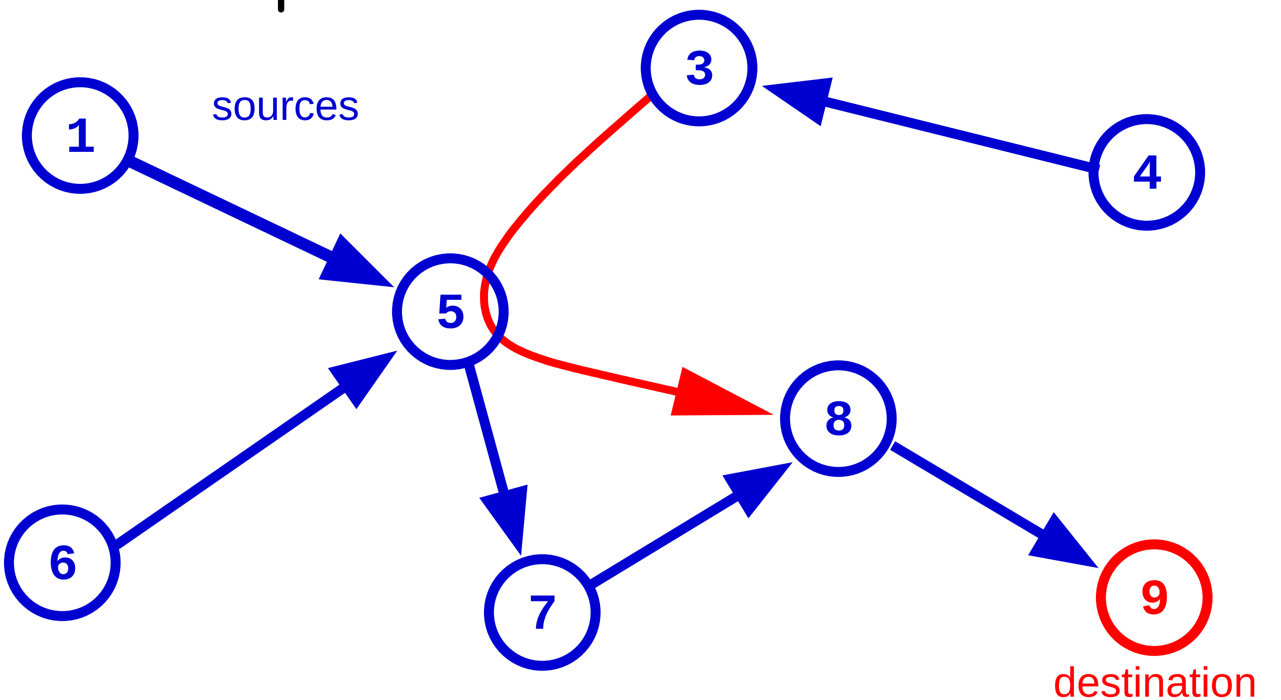


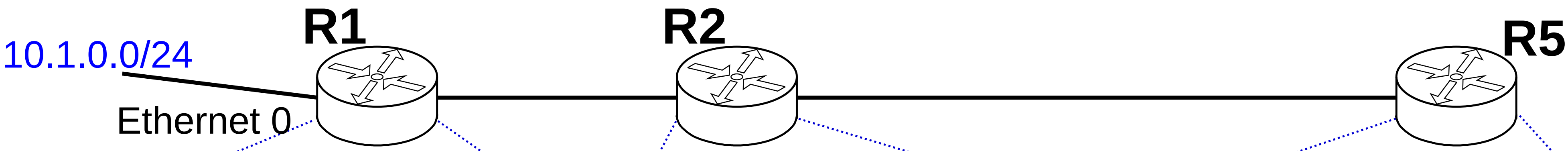
Distance Vector

- also called Distributed Bellman-Ford
- proved converges for shortest path routing
  - ordering and timing of updates doesn't matter
- chief advantages
  - history (RIP invented way back in ARPANET)
  - simplicity
    - example of Cisco RIP configuration

```
router rip
    network 10.1.0.0
```

- problems
  - convergence time (minutes)
  - scaling (of RIP)
  - count to infinity

Count to infinity



R1:

subnet	10.1.0.0/24
next hop	Ethernet 0
distance	1

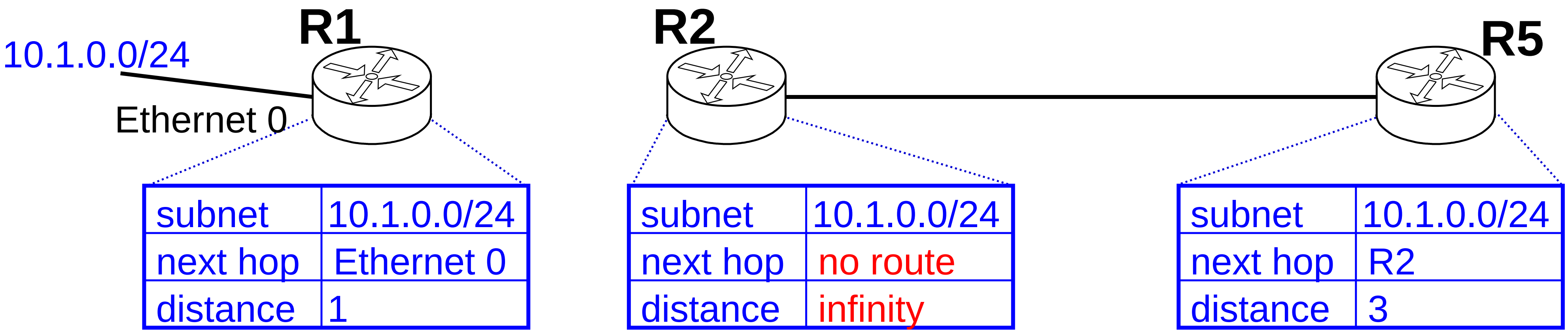
R2:

subnet	10.1.0.0/24
next hop	R1
distance	2

R5:

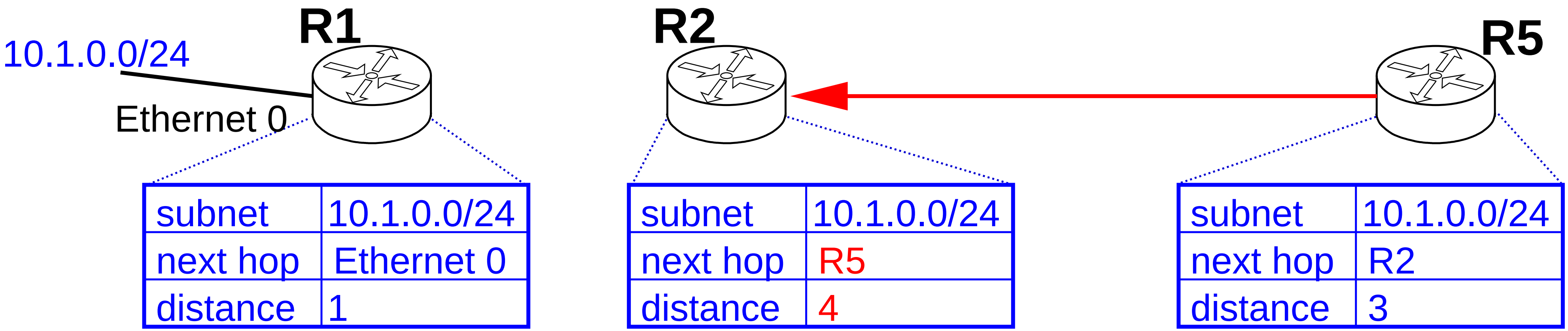
subnet	10.1.0.0/24
next hop	R2
distance	3

Count to infinity



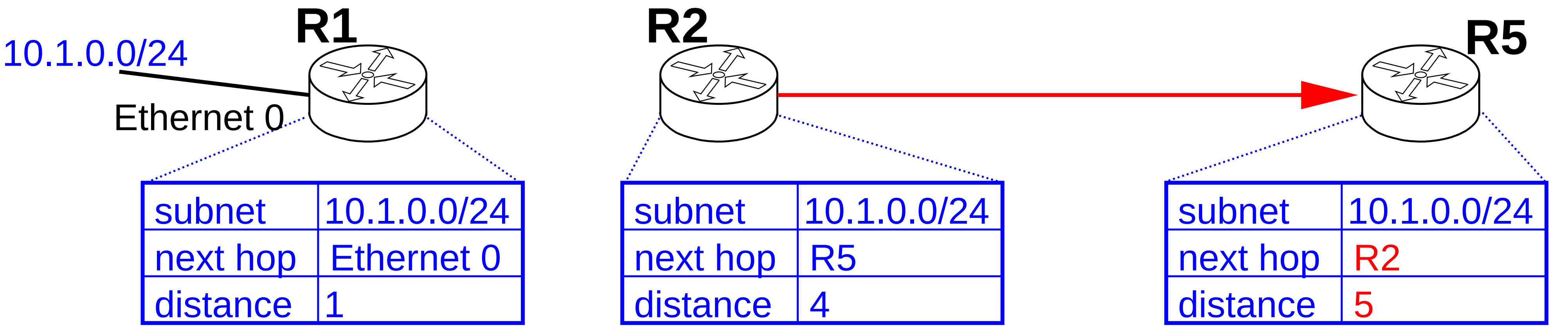
- link between R1 and R2 fails
- R5 does not see the failure!

Count to infinity



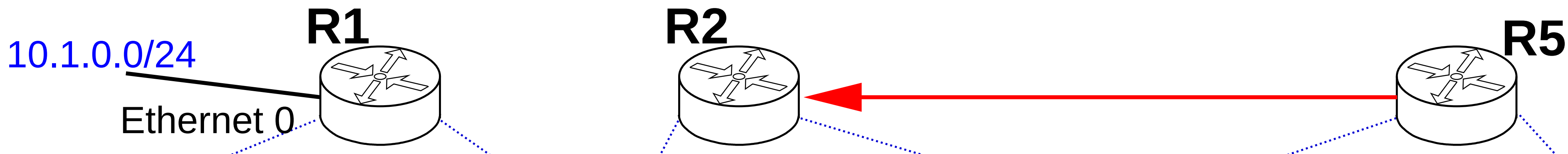
- route update from R5
- R5 does not know that its route is now invalid
- R2 does not know that R5's route is invalid
- a route loop is created

Count to infinity



- R2 does not know R5's route is invalid
- so re-advertises
- R5 sees this as its only valid route

Count to infinity



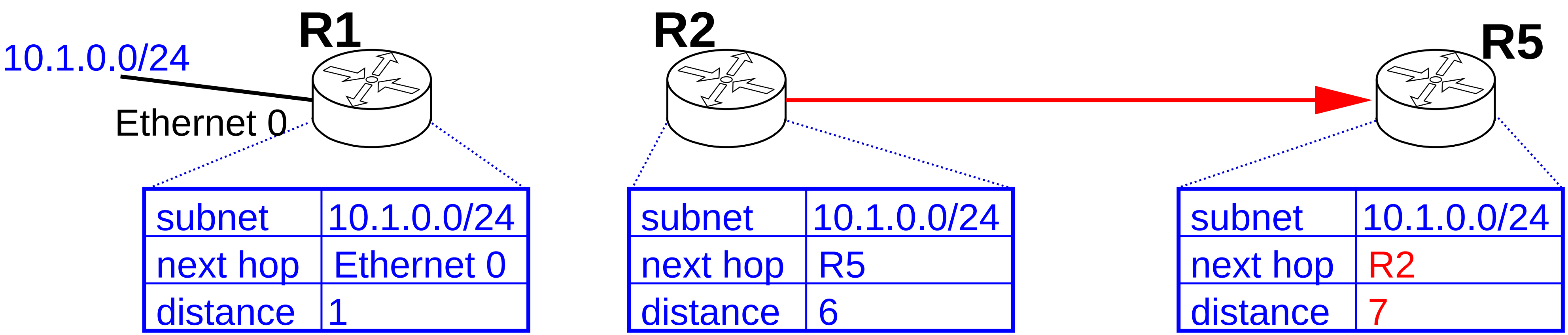
subnet	10.1.0.0/24
next hop	Ethernet 0
distance	1

subnet	10.1.0.0/24
next hop	R5
distance	6

subnet	10.1.0.0/24
next hop	R2
distance	5

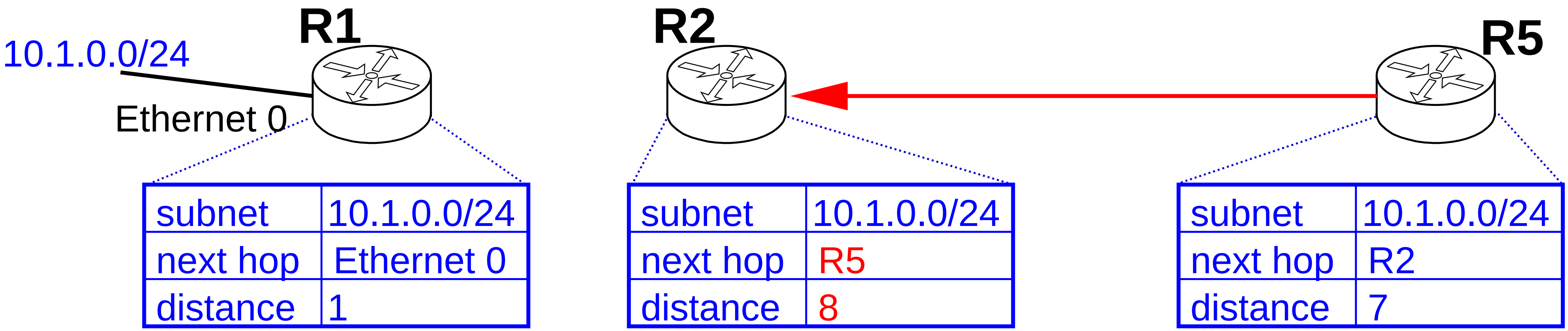
- R5 re-advertises route

Count to infinity



- R2 re-advertises route

Count to infinity



- this process continues indefinitely
- metrics slowly **count to infinity**

RIP

Routing Information Protocol (RIP)

- RIP was first developed in early ARPANET
  - RIPv1, defined in RFC 1058 [3] (1988)
  - RIPv2, defined in RFC 1723 [4] (1994)
    - introduced classless routing (CIDR)
  - RIPng, defined in RFC 2080 (IPv6)
  - MDS authentication RFC 2082.
- implementation
  - uses UDP over IP, on port 520 to carry its data
    - see RFCs for packet formats
  - router transmits full updates every 30 seconds
    - by default

RIP

- count-to-infinity mitigated using
  - split horizon with poison reverse
  - triggered updates
- count-to-infinity stopped
  - maximum distance = 15
  - infinity = 16
- problems
  - convergence is slow
  - count to 16 can still be slow
  - generates lots of traffic
  - maximum length path is 16

References

[1] J. Moy, "OSPF Version 2." IETF, Request for Comments: 2328, 1998.

[2] D. Oran, "OSI IS-IS Intra-domain Routing Protocol." IETF, Request for Comments: 1142, 1990.

[3] C. Hedrick, "Routing Information Protocol." IETF, Request for Comments: 1058, 1988.

[4] G. Malkin, "RIP Version 2." IETF, Request for Comments: 1723, 1994.