Communications Network Design
Class Exercise 3 Solutions

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1. Solve the following problem using the branch and bound algorithm. For convenience, always select x_1 as the branching variable when both x_1 and x_2 are fractional. Show your working (you may use Matlab, or some other tool to solve relaxations, but show the solutions for each branch, and the order of examining them, and explain your reasoning at each step)!

$$\begin{array}{lll} \text{Maximize} & z = x_1 + x_2 & \\ \text{subject to} & 2x_1 + 5x_2 & \leq 16 \\ & 6x_1 + 5x_2 & \leq 30 \\ & x_1, x_2 \geq 0, \text{ and integer} & \end{array}$$

Solution: The problem is illustrated in Figure 1, which shows the values of the objective function $z = x_1 + x_2$ on the integers inside the feasible region. Figure 2 shows the solution via branch and bound. The additional constraints (for the relaxation LP are shown in each box, along with the maximized value of the objective function). The solutions to the relaxation problem are shown outside the boxes, where the problem is feasible. The final solution for the maximum is $z = 5$ at any of the three points $(3, 2)$, $(4, 1)$, $(5, 0)$.

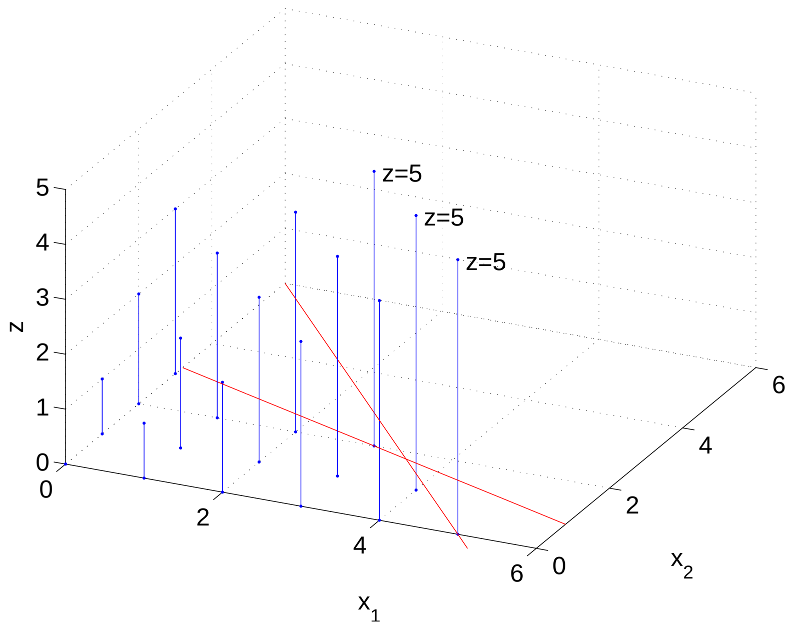

Figure 1: Solution space for the integer program.

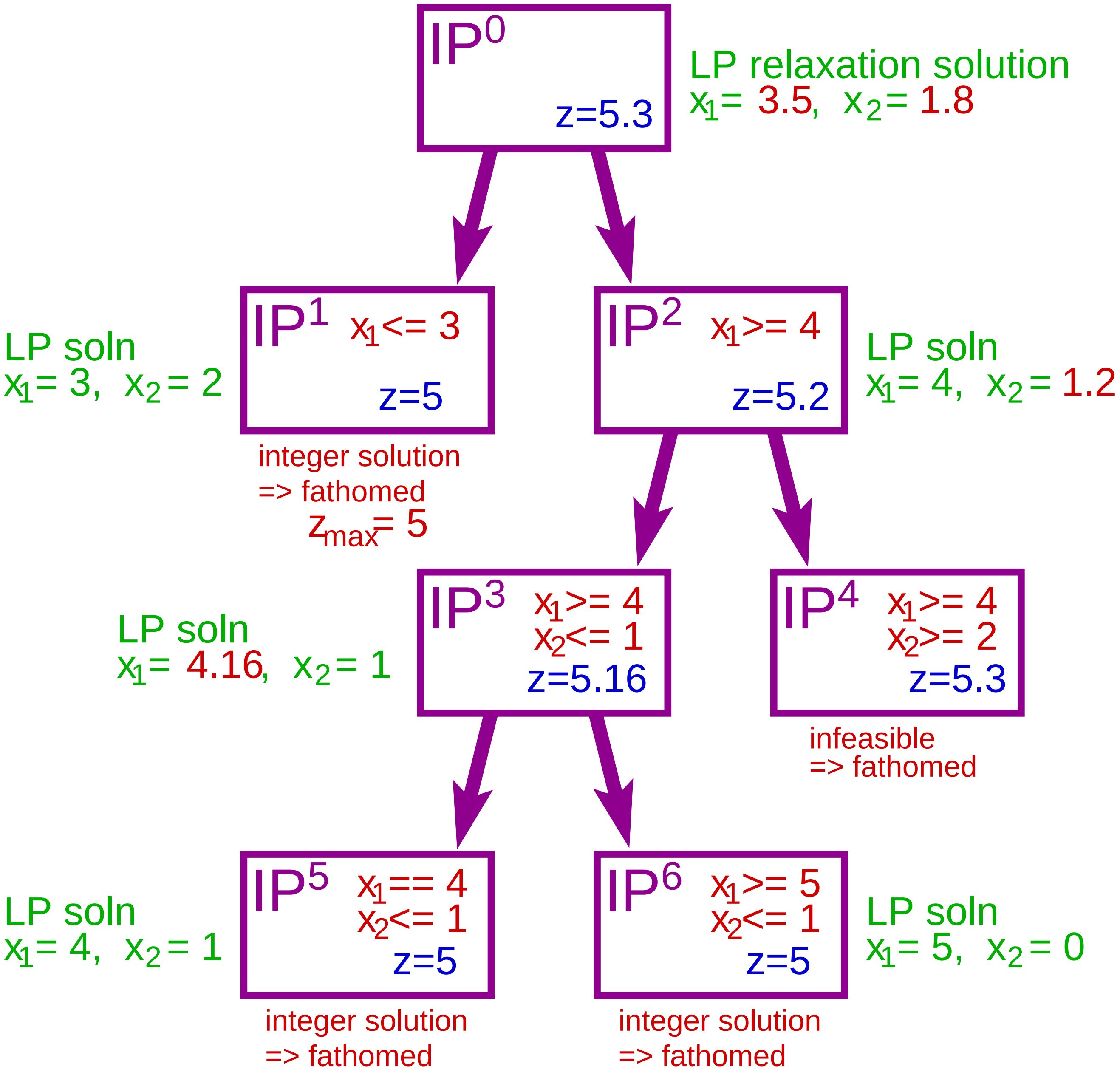

Figure 2: Branch and bound applied to the integer program.

2. B&B solution to knapsack problem with volume limit of 10.

	Item					
	1	2	3	4	5	6
volume	9	4	3	7	1	2
value	6	2	9	3	9	6
value/unit vol.	2/3	1/2	3	3/7	9	3
index	4	5	2	6	1	3

Find the indicator vector z that takes value 1 when an item is included in the pack, and zero otherwise.

Solution:

$$z = (0, 1, 1, 0, 1, 1).$$

which exactly fills the volume (10) and has value 26.

The steps take are illustrated in Figure 3.

3. Find the best solution you can to the following integer knapsack problem with 12 objects with $B = 15$.

	Item											
	1	2	3	4	5	6	7	8	9	10	11	12
volume	5	9	7	7	9	10	5	9	5	1	9	3
value	6	2	2	6	5	5	6	3	7	4	10	6

Solution: My best solutions are

$$z = [1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1], \quad z = [0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 0, 1],$$

both with value 23, and volume 14, branch and bound solution steps appearing in Figure 4. The two solutions are equivalent because items 1 and 7 are identical.

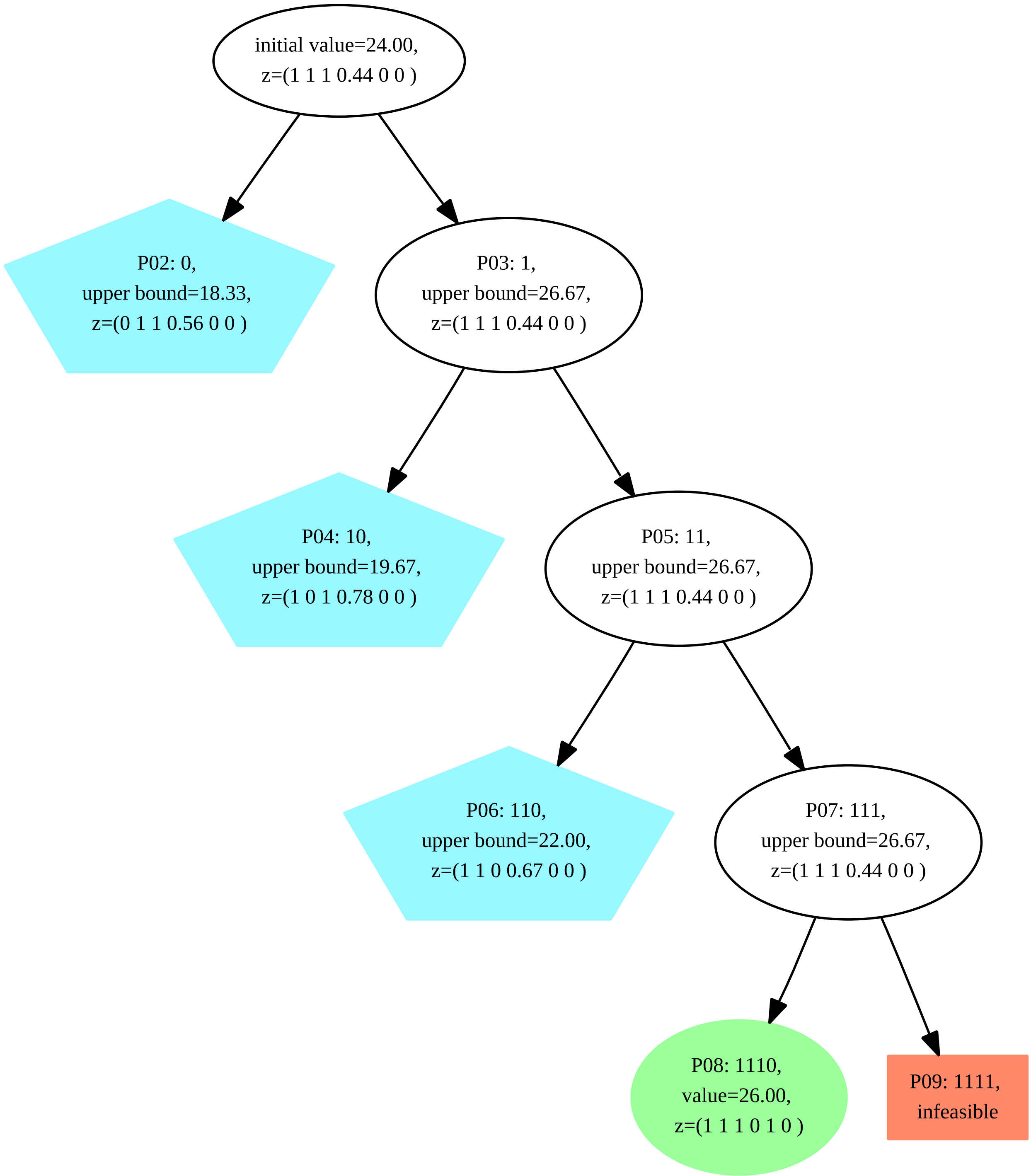

Figure 3: Branch and bound applied to the knapsack problem. Each relaxed problem is indicated by a problem number PN, the values of z_f, the variables we fix, and the lower-bound derived from the relaxed problem. Feasible solutions are shown as ellipses. Fathomed parts of the tree are indicated by the lack of children, infeasible cases are shown as shaded red rectangles, and integer feasible solutions are shown as shaded green ellipses. For instance, problem $P06$ has $z_f = (1, 1, 0)$, and lower-bound of 22, which is less than the incumbent value derived from the initial solution at the top of the tree, and so this branch is fathomed, and further children pruned from the list of problems. Remember that z_f is given in terms of the items sorted by value per unit volume, so these are not directly comparable to the final solution given above.

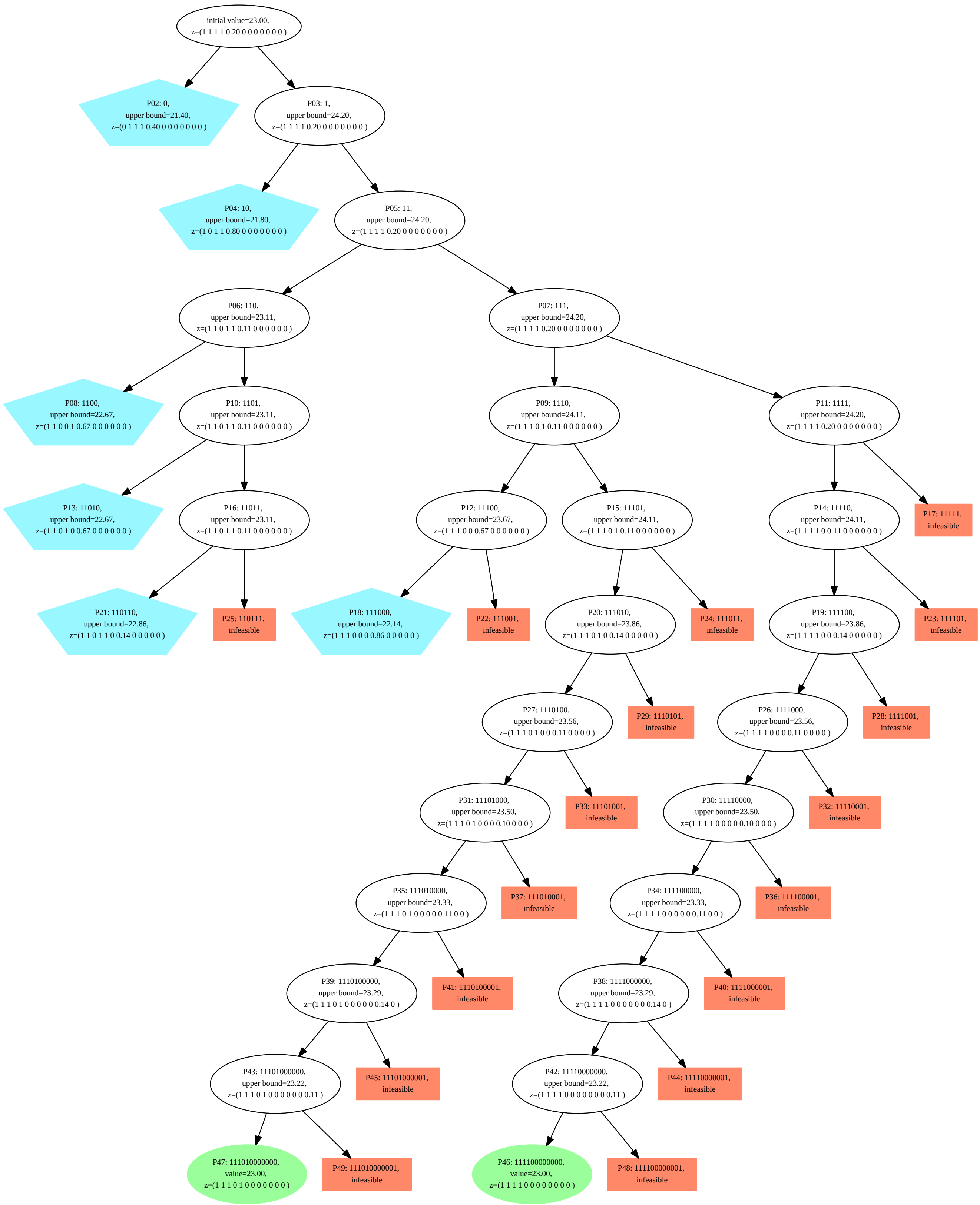

Figure 4: Branch and bound applied to the knapsack problem, drawn as in Figure 3.